Univ of
Toronto
Library

ACTUARIAL SOCIETY OF AMERICA

"The work of science is to substitute facts for appearances and demonstrations for impressions." — RUSKIN.

TRANSACTIONS

1904

VOLUME VIII.

Nos. 31, 32.

With Index.

HG
8754
A4
V.8

NOTICE.

The Society is not responsible for statements made or opinions expressed in the articles, criticisms and discussions published in these "Transactions."

CONTENTS.

	PAGE.
Officers, Members and Associates	iv–vii
Notation	viii
Papers, May 19th and 20th, 1904	1–64
On the Principles which should determine the Maximum Single Risk, and the Acceptance of Exceptional Classes of Risks. Walter S. Nichols	1
Notes on a Factor, hitherto overlooked, of the Rate of Interest. Charlton T. Lewis	8
An Experiment with the Specialized (Mortality) Investigation. Rufus W. Weeks	17
Frequency Curves and Moments. Robert Henderson	30
The Decline in the Birth Rate in New South Wales. Richard Teece	43
Methods of Intervaluation Ascertainment of Reserves. M. M. Dawson	55
Note on Lubbock's Formula. John A. McKellar	58
President's Address, May 19th, 1904	60
Discussion of Papers presented May 20th and 21st, 1903	65
Minutes of Annual Meeting, May 19th, 1904	95
Members and Associates Admitted and Enrolled	96
Fackler Associates' Triennial Prize	96
Obituary Notices, John Bell Tennant	101
James Meikle	102
Arthur Francis Burridge	103
Papers, October 20th and 21st, 1904	105–134
Effect of Total Abstinence on the Death Rate. J. G. Van Cise	105
A Graduation of the Specialized Mortality Experience. A. Hunter	114
On the Repayment of Loans by Contingent Instalments. H. W. Robertson	128
Discussion of Papers presented May 19th and 20th, 1904	135
President's Address, October 20th, 1904	179
Minutes of Semi-Annual Meeting, October 20th, 1904	182
Obituary Notice, Charlton Thomas Lewis	186

ACTUARIAL SOCIETY OF AMERICA.

1904.

THE COUNCIL.

OFFICERS: ISRAEL C. PIERSON, PRESIDENT.
RUFUS W. WEEKS, 1ST VICE-PRESIDENT.
DANIEL H. WELLS, 2D VICE-PRESIDENT.
ARTHUR HUNTER, SECRETARY *pro tempore*.
ARCHIBALD A. WELCH, TREASURER.

EX-PRESIDENTS: DAVID PARKS FACKLER,
HOWELL W. ST. JOHN,
EMORY McCLINTOCK,
BLOOMFIELD J. MILLER,
THOMAS B. MACAULAY,
OSCAR B. IRELAND.

		Term Expires.
ELECTED:	F. SANDERSON,	May, 1905
	J. G. VAN CISE,	May, 1905
	DAVID G. ALSOP,	May, 1906
	WILLIAM S. SMITH,	May, 1906
	JAMES M. CRAIG,	May, 1907
	JOHN K. GORE,	May, 1907

MEMBERS.

Thomas Gans Ackland,
David Griscom Alsop,
Jesse John Barker,
Amédée Bégault,
Alfred Kimball Blackadar,
Samuel Swett Boyden,
Thomas Bradshaw,
John Jones Brinkerhoff,
David Carment,
James Chisholm,
Hubert Cillis,
Henry Cockburn,
James McIntosh Craig,
James Camp Crawford,
Emma Warren Cushman,
Adolph Davidson,
Miles Menander Dawson,
Joseph Arend De Boer,
David Deuchar.
William Roadley Dovey,
David Parks Fackler,
Robert Patterson Field,
Morris Fox,
Frederick William Frankland,
John Marshall Gaines,
William Standish Gaylord,
James Howard Gore,
John Kinsey Gore,
William Joseph Graham,
Paul Guieysse,
Clayton Colman Hall,
Samuel Stickney Hall,
Robert George Hann,
Edward Edgington Hardcastle,
Francis Henry Hemperley,
Robert Henderson,
Augustus Hendriks,
William Hendry,
Charles Daniel Higham,
Charles Hildebrand,
William Richmond Hitchins,
John Marshall Holcombe,
George William Hubbell,
William Hughes,
Arthur Hunter,
Robertson Gilbert Hunter,
Robert Watkinson Huntington, Jr.,
William Anderson Hutcheson,
Oscar Brown Ireland,
Frederick Henry Johnston,
Charles Nelson Jones,
George King,
George Halsey Kirkpatrick,
Christopher Kyle,
James Morgan Lee,
Omer Lepreux,
George Leslie,
Charlton Thomas Lewis,
Louis Linzmeyer,
Charles Alvin Loveland,
John Bodine Lunger,
Emory McClintock,
Thomas Bassett Macaulay,
William Campbell Macdonald,
Henry William Manly,
Léon Marie,
Elbert Pike Marshall,
William Andrew Marshall,
James Weir Mason,
Hiram John Messenger,
Bloomfield Jackson Miller,
Henry Moir,
Walter Smith Nichols,
Joseph Howard Nitchie,

Sydney Norris Ogden,
Percy Charles Herbert Papps,
John Sherman Paterson,
William Brockie Paterson,
Maximilian Heinrich Peiler,
Israel Coriell Pierson,
Gardner Ladd Plumley,
J. C. Albert Quiquet,
Charles Grant Reiter,
Josephus Hargreaves Richardson,
John George Richter,
Hugh Wilfred Robertson,
John Francis Roche,
Gerald Hemmington Ryan,
Howell Williams St. John,
George Ferry Salter,
George White Sanders,
Frank Sanderson,
Edward James Sartelle,
Frederick Schooling,
Ernst Willem Scott,
Herbert Norman Sheppard,
William Simeon Smith,
Edward Lincoln Stabler,
William Thomas Standen,
Samuel Edgar Stilwell,
Wendell Melville Strong,
John Tatlock,
Richard Teece,
Herbert Cecil Thiselton,
Morris Whittemore Torrey,
Joel Garretson Van Cise,
James Douglas Watson,
Rufus Wells Weeks,
George Wegenast,
Archibald Ashley Welch,
Daniel Halsey Wells,
William Joseph Hutchings Whittall,
Asa Shove Wing,
Leonard Joan Wolterbeek,
Arthur Barton Wood,
Ernest Woods,
George Badger Woodward,
Peter Troth Wright,
Walter Channing Wright,
Frank Bertrand Wyatt,
Tsuneta Yano,
Thomas Emley Young.

ASSOCIATES.

Charles Hart Angell.
Isaac Bliss.
James Cornelius Brown.
George Edward Bulkley.
Henry Wright Buttolph.
Raymond Van Arsdale Carpenter.
John Randolph Leigh Carrington.
Lawrence Maclagan Cathles.
Horton Woods Cochnower.
Richard Huntington Cole.
James Douglas Craig.
John Sydney Davenport, Jr.
David Stephens Dickenson.
Herbert Beeman Dow.
John Maynard Emery.
Edward Bathurst Fackler.
Benedict Devine Flynn.
Robert Elder Forster.
James Burnett Gibb.
Menno Snyder Hallman.
Harry Pierson Hammond.
Isaac Smith Homans.
Charles Hughes.
Charles William Jackson.
Christian Jensen.
Henry Nicholas Kaufman.
John Archibald McKellar.
William Oscar Morris.
Douglas Hall Rose.
Edwin Henry Smith.
Walter Newell Stanley.
Abel Travassos.
James Herman Washburn.
Alva Courtenay Washburne.

NOTATION.

ADOPTED OCTOBER 23D, 1890.

RESOLVED, that in the presentation of papers the Symbols of the Text-Book of the Institute of Actuaries, with the additions noted below, be followed when convenient, and that in any event the use of such symbols in other senses be avoided; that the letters denoting commutation columns, according to the system of Dr. Farr, customarily followed by American Actuaries, be printed in plain "full-face" type; and that otherwise there be entire liberty as regards notation.

$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}$.

$u_x = \frac{l_x}{l_{x+1}}(1 + i) = \frac{D_x}{D_{x+1}}$.

S = Sum insured.

K = Cost of insurance = $q(S - V)$.

Π = Single premium (alternative for A).

VOL. VIII. No. 31.

TRANSACTIONS.

MAY 19TH AND 20TH, 1904.

ON THE PRINCIPLES WHICH SHOULD DETERMINE THE MAXIMUM SINGLE RISK, AND THE ACCEPTANCE OF EXCEPTIONAL CLASSES OF RISKS.

WALTER S. NICHOLS.

Mr. T. E. Young, former President of the British Institute of Actuaries, devotes a chapter of his recent work on insurance to a consideration of the principles which should determine the limitation of the maximum single risk and the acceptance of special classes of risks different from those ordinarily written. Mr. Young, I believe, expresses the views which are held by some other actuaries on these points, but which seem to me tainted with certain fallacies. He correctly states the question as follows: "The question confronting the company at its establishment and at any subsequent stage, is the maximum amount that may be safely retained upon an individual life without the probability of inconvenient fluctuations in the incidence of its claims." He adds: "It is inexpedient to incur a risk whose discharge would prove embarrassing, impossible or ruinous to meet." This maximum amount, according to Mr. Young, will depend upon the expectation of the company, "Of the number of policies it will probably receive each year for sums equaling or exceeding the amount that may provisionally be determined as the limit of retention." That is a company having

decided upon a maximum line of $3,000, and having all its policies within that limit, in order to increase that maximum line to $10,000 or any other figure must have a reasonable expectation of securing enough of such $10,000 policies to furnish an independent average. This is a familiar doctrine, not only in life insurance, but in other branches as well. The only other legitimate principle governing this increase which he recognizes is the gradual growth of the reserve which by reducing the actual amount at risk will allow an additional sum to be placed on the same life.

Further on we are told that since the necessary number of maximum policies cannot be obtained at once the question of the least number with which it will be prudent to commence is answered by the element of selection. The limit should be so fixed that sufficient number for an average may be confidently anticipated before the five years of favorable selection has expired; otherwise the limit will need to be reduced. As if to leave no doubt of his views on this subject Mr. Young sums them up as follows:

"It is evident from these considerations that the true basis for assessment of the limit of risk does not consist of the extent of the premium revenue or the magnitude of the assets which the company may possess, but solely relates to the number of policies of maximum amount which the nature and range of its business entitle it to expect to secure."

It is because these views of the distinguished ex-president of the British Institute represent those which so widely prevail both among life and fire underwriters that I have quoted so liberally from his work. If he is correct a company, no matter how many policies for $10,000 or under it may have on its books, or how large its assets, would not be justified in writing a single line for $50,000, or any other excessive amount, unless it had a reasonable prospect of securing enough of such policies to maintain an independent average.

The fallacy of this doctrine will be seen in a single illustration. Let us assume five companies with maximum lines of $5,000 and alike in all other respects, but whose business is such that any increase of the limit might result in the inconvenient fluctuation which according to Mr. Young is to be avoided. Since they are independent companies their average is in no way

disturbed by the granting of $5,000 policies by each upon the same individual life, or upon half a dozen such lives. Now let those companies agree to amalgamate into one. It is obvious that their risk of fluctuation has been reduced by the combination, and that their profit earning and dividend paying abilities have been in no way disturbed. So far as the probability of inconvenient fluctuations in the incidence of its claims is concerned the aggregate corporation is more secure than were each of its separate factors. Yet if the principle laid down by Mr. Young were correct, the $5,000 risks on the same life, which were entirely proper when distributed among the independent companies, became a disturbing factor the moment they were combined into one. Or to put the case in another form. Assume a company with sufficient risks of $50,000 each to average among themselves. According to the theory of Mr. Young, if of all those risks but one or half a dozen were exchanged for ten times the number of risks of $5,000 each, the one or half dozen risks remaining would open the door to serious danger of fluctuation. As a matter of fact, on the contrary, the risk of fluctuation would be reduced to one-third of its original amount by the change. The mere statement of the case in this form is a refutation of the theory. The limit of the maximum line, instead of being dependent on obtaining sufficient policies of any particular class and being independent of the resources and general business of the company, depends almost solely on the latter and the company may be justified in accepting a single large policy though unable to secure another. As has been well remarked to the writer by a member of this Society, the best test of a proper limit is the strain liable to be put upon the surplus by fluctuations. Hence it is to those elements of the resources involved in the existing and expectant surplus that we must largely look for guidance. The principle involved is but another phase of the familiar doctrine of moral hazard. The capitalist of large resources can profitably engage in speculations which would be dangerous to the man of moderate means. He may profitably make a single venture which would be folly on the part of one having but limited funds. The maximum line which any corporation can profitably write is largely determined by the least number of policies into which its aggregate risks could be divided and still furnish a satisfactory average.

Beyond insisting that the maximum policies must be sufficient to average among themselves the author quoted does not attempt to lay down any fixed rule for determining the limit. This is manifestly impossible since it depends wholly on the risk of fluctuation which a company may deem it prudent to assume. There are several factors entering into this question which are apt to be overlooked. The danger of serious fluctuations from an average largely depends on the frequency with which a company must balance its accounts. The corporation which is compelled to make annual returns and apportion its profits each year may be seriously disturbed by a fluctuation which would be of no moment if such balance were only struck at longer intervals, and must consequently reduce its maximum limit according to the frequency of the intervals within which it must render an accounting. Time is an element hardly less important than the number at risk in reducing danger of fluctuations. One of the serious drawbacks encountered by the fire underwriter to-day is the fact that except with the largest companies the experience of a single year is insufficient to furnish a reliable average, while the company must render an annual statement of its affairs.

The actual amount of fluctuation which a company may treat as normal and which determines the maximum line which it may write is not a mathematical question at all. It is like the safety co-efficient of the engineer or the modulus of the mathematician, an arbitrary quantity which is governed by purely business considerations. The temporary excess of death losses or diminution of profits which a company can meet without injury to its standing or business is the proper limit to its normal fluctuations, and this is regulated by the business sentiments or prejudices of its members and of the public.

Another factor in the risk of fluctuation is the actual percentage of expected loss. The smaller the probability of death, other things being equal, the larger will be the deviation from the average among a given number of risks. The higher the premium rate the fewer the policies required and, wholly apart from the increase of its reserve, a company may expect a more uniform experience as its policies grow older. This is well illustrated in marine insurance where steamship lines controlling a limited number of vessels frequently find self-insurance profitable, a thing which they would hardly venture to undertake if the risk of loss and consequently the cost of the insurance were less.

Still another important factor in the risk of fluctuation is found in the nature of the causes themselves which produce loss. The mathematical theory of fluctuations is founded on the principle of least squares, and as is well known treats the causes as independent and infinite in number and equal in efficiency. The law of probability involved is the same as that which determines the relative sequences and distributions of heads and tails in an indefinitely large number of tosses of a penny. The actual laws of causality that are operative in the physical world are very imperfect counterparts of this mathematical law. The causes are irregular or periodic as in the case of epidemics. They are often interacting and cumulative as in the case of contagions. They are unequal in efficiency and varient both in their form and character. The tendency of a large proportion of physical causes is to operate by rythmic impulses and to create partial and temporary means which are themselves fluctuations from the average. This feature is seen in the sharp alterations of temperature which accompany the changing seasons, and in the successions of storms through which the static equilibrium of the air is restored. The past winter has furnished a noted illustration in the shifting of the barometric pressures that were responsible for its exceptional severity. Each class of risks has its special causes which determine the character and extent of those temporary deviations from the mathematical law referred to and consequently the requisites for a safe average. The fluctuations to be expected by a fire company are vastly greater than those of a life company though the cost of insurance and the number of risks might be the same. The fluctuations in our inland marine writings are so great that no single year's experience can approximate an average and the business has at times degenerated into a mere gamble. The rule holds true in a minor degree among the various classes that make up the risks of a life company. Every transparent medium has what is called its own special index of refraction which determines how much a ray of light will be bent in its passage from the normal. So every class of risks may be said to have its peculiar index of refraction which will determine the temporary departures from the mathematical law of error which may be looked for during a stated time. It will be evident from these considerations how complicated is the problem involved in a maximum line even when the

actual cost of insurance is assumed to be known and we have only to deal with probable fluctuations from a normal average. Any uncertainty about the cost of insurance itself introduces new and still more complex considerations. Here we meet another class of risks which are also discussed by Mr. Young. Such are the risks on rated up or inferior lives, on negroes, on those exposed to special dangers from occupation, residence and the like. Indeed it may be assumed to exist in respect to each of the numerous classes into which the ordinary risks of a life company may be divided by virtue of their occupation, residence or other peculiarities, and even to extend to the individual peculiarities which distinguish one policy-holder from another.

Uncertainty regarding the actual cost of insurance must be dealt with on principles in some respects different from those attaching to the mere risk of fluctuations. Instead of seeking for enough of a certain class to average among themselves, it may be necessary to minimize the possible loss by limiting the number and amount of any specific class which shall be taken, and to seek an average by offsetting the uncertainties of one class against those of another. This is an important fact to be borne in mind in connection with all insurance ventures where sufficient experience has not been secured for trustworthy knowledge of the actual cost of insurance. It is notoriously applicable to casualty insurance where risks must often be assumed on a limited experience or perhaps none at all. Instead of such companies confining themselves within narrow lines as is often insisted on, their best protection is often secured by a variety of uncertain hazards, in which the errors regarding one class may be offset by those regarding another. It enters in some measure into every form of insurance no matter how wide the experience. No company can do more than approximately estimate the cost of carrying the business on its books, or tell how closely it may resemble the standard table from which its rates were computed. In fire insurance where the hazards are continually changing through economic or structural causes, serious losses have been entailed in many instances by making specialties of certain classes previously profitable. In a last analysis each individual risk involves peculiarities which differentiate it from every other and contribute to this uncertainty as to the cost. So that the aggregate risks of an ordinary life company may be regarded as

compensating the possible errors in their individual cost of insurance by offsetting the one against the other through differences in age, occupation, residence, constitutional features and the rest.

The practical effect in dealing with any uncertain classes of risks such as under average or tropical lives is that they can only be profitably taken in limited amounts as an experiment until a satisfactory experience has been secured, or else at rates sufficiently ample to overcome the risk of error. To depend on an average to offset the danger would be a serious mistake. The correctness of this principle is recognized by Mr. Young and the reasons for it are clearly set forth in his book. But the very logic of his argument if applied to ordinary fluctuations from an average which are after all but a special and limited case of error in the cost of insurance would have led him to see that the maximum line depends on the general business of the company rather than on securing enough of a special class to average among themselves.

A small volume might be written on the mathematical principles involved in this complicated subject. I have attempted to touch only on a few leading points in the belief that the issue which I have thus ventured to raise with the distinguished ex-president of the British Institute, and with such actuaries as concur in his views, may afford an interesting topic for discussion.

SUPPLEMENTAL.

Since this paper was written, I have received from Mr. Young an exceedingly pleasant communication called out by a short article on the same subject in a recent issue of the *Insurance Monitor*. Mr. Young informs me that he can perceive no difficulty in accepting the views which I have expressed in the earlier part of this paper, but that in writing he had specially in mind those important fluctuations in mortality which related essentially to bonuses and were significant simply in that aspect. Had his treatment of the subject been more extended it would probably be found, he says, that "little if any difference of judgment exists between us." I, in turn, would probably have little difficulty in subscribing to the views of Mr. Young when interpreted as he apparently intended and restricted to the case of a company whose dividends were threatened with disturbance by an excessive fluctuation in the mortality.

Notes on a Factor, hitherto overlooked, of the Rate of Interest.

CHARLTON T. LEWIS.

The instinct of the ant and of the squirrel to hoard food for winter keeps their species from dying out. In man, foresight and provision for the future take a far wider range; but the propensity to save is almost as instinctive in him as in the dumb creatures. Some people have it as a ruling passion; others are without it; and the difference has no constant relation to other differences of character or mind. Men great in thought and action are often destitute of the impulse to accumulate, while others, with endowments in general of a low grade, are insatiate in hoarding. The disposition, however, is second to no natural gift in helping to success in life, as success is commonly measured. All wealth is the result of saving. This practice, whether a religious duty or not, is a social and economical virtue, and the decalogue has no precept which is more plainly a "commandment with promise." Young people who lack the saving instinct, then, ought to be taught and trained in the saving habit, and this is certainly nobler if acquired by reason, and practiced with intelligence, than if indulged as a blind impulse or a passion. Prudence claims esteem, however greed may deserve contempt.

To save with intelligence, one does not need a profound knowledge of economics, but must understand some elementary principles of social science, and certain laws which can be precisely expressed only in an arithmetical form. A little patience, however, will make the essentials of this learning very simple, and will discover in it great practical interest. Mere hoarding is efficient only for savings which will keep till the foreseen time of need. This fact limits the savings of bees and of savages to provision for short periods. The surplus products which can be spared from to-day's consumption will keep till to-morrow, or till next winter, but are too perishable to be trusted for a more remote future. Civilized man has a vast system of division of labor, of barter and of exchange, and of money; the

grand result of which is the conversion of savings into capital. The word capital, which every man in the street utters glibly and reasons about with no suspicion that he does not conceive it definitely and understand it clearly, is really a complex and elusive term. By its vagueness and its numberless shades and facets of meaning, it becomes the source of confused thought in logical forms, and the support of many fallacies and heresies in economics and finance. For our purpose, it is enough to show that while it is commonly thought of as a group of real things, an aggregate of possessed objects, its true meaning is almost opposite of this. It is essentially a continuing claim upon or right in an ever changing mass of things; it is a power, at any time, to appropriate a definite part of the wealth that is perpetually produced and consumed and reproduced by the whole community.

Property consists of individual things, each of which can be seen and handled, used and enjoyed; each with its own distinct features of utility. Whether these are acres, houses, horses, machines, books, or loaves of bread, no two are precisely alike, and there is a choice between them. Capital consists of dollars or other ideal units of an infinite mass, every one of which is absolutely identical with every other. Property is fixed in place, each part with its own definite use and purpose, and undergoes the changes impressed on it by labor, or is consumed and disappears. Capital is in perpetual flow, suffering transitions of place and changes of form as swift as thought, with no impairment of its integrity or loss of value. Its evidence or symbol may be in your pocket or on your bank book in the morning, and at your will, before noon, it shall take the shape of a share in the yield of a Klondike gold mine or an African diamond pit, an income for nine hundred years from a New York railroad or a right to a fixed payment ten years hence by the Emperor of Japan, an assured endowment for your old age, or a provision for your family when you die. All these differences are explained by the fact that property is the physical product of labor upon material things, while capital is the creation of mind. Property is the body, capital the soul of industrial life, and the measure of economical civilization is the degree in which property has been co-ordinated, collected, idealized and energized by conversion into capital, or rather by its expression and control in the form of capital.

Now it is this opportunity to make capital of whatever can be produced above current needs that elevates hoarding into saving and makes possible a provision for an indefinite future. But capital, as soon as it begins to exist, comes into demand. Being convertible at once into control of whatever utility its holder most needs for enjoyment or production, its use is valuable and will be paid for. Hence it commands interest; that is, men are ready to take and employ it, and afterwards to return it, increased by an addition given for the privilege. The power of interest is the key to the accumulation of capital, as distinguished from the production of property. In a less advanced social state, rental and hire are paid, now in kind, now in whatever serves as money, for the loan of every kind of property. But the lender gets what he can and the borrower pays what he must, according to the circumstances of the particular case, such as the urgency of the need, the extent of the supply, and the skill of the parties in bargaining. Just in the degree in which all property is fused into homogeneous capital, whose parts being interconvertible are open in competition to all borrowers, is there a tendency to uniformity in the price paid for its use. A current or market rate of interest tends to displace the extreme irregularities of individual contracts. This movement, while it indefinitely increases the efficiency of capital in industry, vastly diminishes the average rate paid by borrowers. It is the chief agency in regulating and limiting interest.

A library has been written on the nature of interest, and the forces which determine its amount. The discussion is not closed, and the theory of the subject is still in confusion. No economist has as yet grasped all the conditions of the problem so as to reach a solution which accords at every point with experience. But among these conditions are certain general facts of history, too much neglected, which serve to confute doctrines often advanced as scientific and accepted truths. Thus it is certain that the increase of wealth does not necessarily lower rates of interest; that there is not a progressive and secular diminution of these rates as the world grows richer and industry more productive; and that in particular the growth of the stock of gold, and of its substitutes, in use as money, has no tendency to make the use of capital cheaper. These facts, which seem paradoxes to popular writers on finance, and even to many economists, are established

beyond dispute by centuries of experience, and any theory which contradicts them is narrow and incomplete. Periods of discovery and invention, of active enterprise in industry, of sanguine hope and wide speculation, are those in which wealth grows most rapidly, and in which the demand for capital, over-reaching the supply, forces the rate of interest upward. Such periods alternate with those of comparative quiet and stagnation, in which the cautious and slow accumulation of capital gradually overweights the market demand, and depresses the rate. An enlargement of the supply of money in circulation stimulates speculation, and creates a demand for loans which is partly artificial and usually excessive, and often drives the price of them too high for productiveness to bear, until a commercial crisis breaks the speculative bubble, and sluggishness and depression recur.

In short interest, like every other inquiry in Economics, offers a problem, not so much in the nature and properties of things, as in the nature of men. The so-called "dismal science" is made dismal and fruitless by narrowing its view to its materials, and losing sight of the varying and contending forces which move them. Since people will always seek their own advantage, we quietly assume that they will act for that which we know or believe to be for their advantage. But the deviations from this rule, which the science of selfishness would regard as exceptions, are far more numerous than the instances of it, and cover the most important parts of human activity. Were it not so, half the purchases, nine-tenths of the wars, and ninety-nine one-hundredths of the speculative investments, made by men, would be unknown. Habit, prejudice, imperfect knowledge, sudden impulse, waves of sentiment, sympathy with associates, sanguine hope, rash trust or excessive timidity, affect men's views of their interests, and thus modify all economic conduct and control large portions of it. If prudence were the supreme factor in practical life which some economists assume it to be, the world would be richer, duller and less worth living in than Holland was a century ago.

These considerations may satisfy us that interest is a fluctuating, not a gradually vanishing fact. History proves that the rate at which savings may confidently be accumulated varies under many influences, chiefly the varying energy of enterprise and speculation; that in a progressive civilization and

during peace it is not likely to rise above six per cent. a year, nor to fall many years together below four; that taking into view long periods and large market areas, it tends to compound with the average yield expected from capital in productive industry; that the latest of several well defined secular waves of change in the rate of interest, covering the civilized world, began to rise about 1845, and continued on the whole to advance until about 1872; that a retrogression then set in which lasted until five or six years ago; that the prevailing rates were then at the lowest, and have since slowly increased, with impressive indications that the tendency will be upwards for many years; and justifies the assured expectation that investments, as safe as the uncertainties of human affairs permit, with a yield of four per cent. every year, will be available to the prudent investor through the next generation.

But this expectation, it must be insisted, rests upon the nature of man in a progressive society, and not upon the nature of property. Were the spirit of enterprise destroyed and the speculative hazards of fortune ended, the demand for capital in industry would be limited to the amounts needed under old and tried methods of production. The promise of yield would be no greater than the average additions to wealth habitually made by such methods. The element of expectancy, the sanguine hope which is the inspiration of enterprise, would lose its power to lift the rate of interest above the general rate of actual accumulation, and interest would fall to a lower point than it has ever reached in the modern industrial world. No such change in human nature is probable, but that it is at least conceivable is shown, not only by the example of the Roman Empire in its decline, but by that of Spain and Holland in the eighteenth century. And the conception deserves to be carefully defined, and vividly realized, because it teaches a practical lesson of value. The level about which the rate of interest fluctuates, and to which apart from disturbing influences it always tends, is the rate of yield which enterprise expects from its use in industry. But this rate is always materially higher than the average rate of increase which industry actually obtains by its use.

This truth admits of a simple and complete demonstration. It is four hundred and twelve years since the discovery of America. The average rate of interest on this continent

throughout that time has been more than six per cent. But if one dollar had been saved each year, invested at that rate, and accumulated until now, the fund would amount in 1904 to four hundred and forty-seven thousands of millions of dollars, or nearly five times the aggregate estimated wealth of the inhabitants of America. Were the rate only three per cent., the sum would be twelve thousand millions of dollars, a multiple of the world's stock of gold. There is no real measure of a nation's wealth; the application to it of the dollar or other unit of money, in census appraisals or statistical estimates, is meaningless. But it is certain that the entire capital of the people has not at any time increased at the larger rate named, and it is probable that the lower rate of growth has been reached only recently and for short periods of exceptional prosperity. With the population growing by two per cent. each year, if we can suppose that their wealth accumulates at the rate of four per cent., then in three generations the average command of capital held by each citizen will have multiplied eight-fold, and the community will be by far the richest that has ever existed on the earth. These brief hints are enough to show that, in our progressive civilization, the buoyant hope which inspires enterprise is a continuing force which maintains the rate of interest on capital far above its actual net productiveness.

Such considerations furnish a complete refutation of some plausible economic theories. The late Henry George supposed himself to have modified and completed Ricardo's theory of rent by discovering that the proprietor of land has no right to the added utility given it by neighboring improvements and by the concentration of population and trade around it. A logical analysis of the conditions of the question would prove that one's original appropriation of land secures to his successors, by assignment or inheritance, precisely the same title to its occupancy and use, together with all modifications of its value wrought by labor or circumstance, as a similar appropriation of any other gift of nature. But the stress of his reasoning lies in the supposition that land, because of its absolute limit of quantity, forms a monopoly, and naturally increases its value more certainly and more rapidly than any other form of property, and that through agencies wholly independent of the holder's care or labor. This argument withers before the historical fact that

there is not in the world a considerable body of land, which for a long period of time taken together, has increased in market value at a rate comparable with the current rate of interest on movable capital. The single-tax theorists take their typical and extreme instances of "unearned increment" from the Island of Manhattan; yet there are not many parcels of vacant land even there the market price of which is as much as the amount of the taxes and assessments already paid on them, accumulated at prevailng rates of interest. And it is certain that the added values of all the land in the United States at this time, at a full appraisal, amount to but a fraction of the value, thus accumulated, of the labor and capital which have been expended on its improvement. This answer, however, like the theory itself, is superficial. A truly scientific inquiry would recognize in what is called the "unearned increment" a social phenomenon; the joint product of all occupants of land and other natural gifts in the community, and a product the distribution of which among them may properly be determined in advance by that of the land.

The practical lesson for investors, which is suggested by these observations, is that, on the large average, the best investments for savings are in capital commanding interest, rather than in specific property yielding hire or rental. Aside from personal earnings, these are the two great divisions of income; and while large profits are often made by enterprise and skill combined with capital, and while great estates have been built up by sagacity and good fortune in acquiring, improving and hiring out landed property, yet for every success in each of these classes of undertaking there is a plurality of failures. The amount of genius, intelligence and industry given to the direct management of capital for its productiveness is incomparably greater than that devoted to its investment in loans, yet the average net income derived from it by the latter process is much larger than by the former. The most brilliant prizes of the industrial world, indeed, are reached only by the hazardous path of enterprise, but the wrecks and losses by which that path is strewn sadly reduce the general average of success. Men of exceptional capacity or management, the born discoverers and captains of industry, will find the career for which nature destines them; but the mass of men whose aim is to save what they can from present needs, and to make it as fruitful as possible

for the future, will do wisely never, without controlling reasons, to deviate from the policy of investment in secured loans, steadily compounding in some form the interest they yield. However fascinating the promise of gain may be in the expected appreciation of values, the comprehensive view proves that the purchasers of specific property for investment, as a group, are far less wise and less successful than the investors in loans, and that the advantage of the latter class is sure to continue so long as sanguine enterprise and hopeful progress characterize the community.

The scientific aim of these remarks is to supply a defect in the received theory of interest. That theory, as indicating the influences which determine the rate, is summed up by the New International Cyclopædia, which represents the latest economic authorities in these words: "The rate of interest depends upon the relation between the demand and the supply of capital. The demand for capital varies with the opportunity for its productive employment; the supply varies with the readiness of individuals to postpone consumption to a future date. If productivity is high, the inducement to save is correspondingly great; and consequently the supply of capital increases. With increase in supply of capital, its productivity declines, until no reason exists for further creation of capital. The rate of interest, therefore, tends to become fixed at the point where the productivity of capital is just sufficient to compensate those who save for the disadvantages which attend the postponing of consumption, or waiting."

This theory is defective in taking no note of the vast difference between the productiveness actually experienced, in the use of capital, and the productiveness anticipated by the manager, manufacturer or speculator, in the use of the capital he seeks. Economists have long perceived that periods of invention, discovery and enterprise are those in which the demand for capital is effective and interest high. But the connection between the spirit of enterprise and the increased demand has always been sought in the slow process of absorbing capital in new enterprises, converting the floating supply into fixed forms, and reducing the available stock in the markets. In reality the connection is far closer and the effect upon the rate of interest is much quicker, than this process can explain.

The reason is that the demand is determined, not by the experience of past productiveness, but by the hope of future profit. Experience is one factor in inspiring hope, but by no means the most general or the most powerful. It is proverbial that the most sanguine adventurers in industry and in speculation are not the most experienced in business. The excess, in the great aggregate, of expectation above realization is a constant fact in the modern world of production and trade; and since it is always the expectation, not the realization, that determines the demand for capital, the rate of interest, under all fluctuations, maintains a level materially higher than the average increase of capital.

This observation, as clearly entitled to be called an economic law as any generalization of its class, is a typical instance of the fundamental truth in economics, that psychological conditions can never be neglected in estimating the effect of human actions on the production, distribution and consumption of wealth. Another instance, of like importance, is the general tendency, when riches increase, to a vast extension of useless and wasteful consumption; a phenomenon which also contributes in a degree, though indirectly, to advance rates of interest.

An Experiment with the Specialized Investigation.

RUFUS W. WEEKS.

The recent Specialized Mortality Investigation has presented to the actuarial world a vast mass of valuable material arranged in systematic form. Various lines of inference and experiment are suggested by the mere possession of this material. One of these lines concerns the matter of a standard for measuring the mortality in the various classes contained in the investigation.

Doubtless the ideal standard would have been a set of mortality tables for successive years of insurance, representing the entire experience of the companies during the thirty years, including the classes actually investigated as well as all the other insurances issued and in force within the same years. This, however, would have meant handling something like five times the amount of material included in the task actually undertaken; so colossal an undertaking was never contemplated for a moment by the Society or by the Committee. The Committee used, for the purpose of measuring the degrees of mortality, an artificial standard, corresponding to the impressions of its members as to what the normal mortality rates among the average of American insured lives have been; and upon this standard are based the published ratios of actual to expected deaths in the respective classes.

Another possible method of treating the matter of a standard for comparison will suggest itself, upon a mere restating of the main facts of the investigation in the form of a list of the classes, arranged in the order of their mortality-quality on the standard of the Committee; a list in which the best class is put first, and the others follow successively, ending with the worst. Such a list is given below. In preparing this list classes seventy-seven to ninety-eight, which consist of persons insured in certain selected counties of the United States, have been omitted.

List of Seventy-six Classes.

Arranged in Order of Committee's Percentages of Actual to Expected Death.

Class	Description	Committee's Expected Deaths	Actual Deaths	Percentage
44	Has pulse below 60	622	436	70.1
18	Potter	48	37	77.1
51	Has had otorrhea, or running from ear	245	195	79.6
20	Printer, not publisher	972	775	79.7
26	Traveling salesman	2,223	1,780	80.1
35	Railway mail clerk	81	65	80.2
13	Exposed in occupation to electricity	258	209	81.0
66	Weight C, parent dead below 70 of kidney or Bright's disease	1,796	1,475	82.1
54	Has had inflammation of bowels, peritonitis or appendicitis	1,237	1,030	83.3
76	Parent, brother or sister has been insane	1,577	1,315	83.4
57	Has had dyspepsia, with weight A, B or C	4,221	3,528	83.6
67	Weight C, parent dead below 70 of heart disease	3,858	3,269	84.7
19	House painter	616	526	85.4
65	Weight C, parent dead below 70 of consumption, phthisis or tuberculosis	6,198	5,382	86.8
14	In sawmill	339	295	87.0
43	Has intermittent or irregular pulse	274	240	87.6
21	Tailor	1,598	1,405	87.9
70	Weight C, both parents reached 75	9,060	8,020	88.5
75	Parent, brother or sister died of cancer	5,279	4,680	88.7
22	Butcher or meat dealer	2,050	1,822	88.9
36	Railway express messenger	38	34	89.5
16	Puddler, foundry or rolling mill worker, etc., not machinist	748	671	89.7
41	Pilot	67	61	91.0
72	Weight D, not included under 58 or 71	13,602	12,452	91.5
58	Has had dyspepsia with weight D	824	775	94.1
68	Weight C, parent dead below 70 of apoplexy or paralysis	6,318	5,945	94.1
6	Born in Sweden or Norway	1,238	1,174	94.8
27	Cattle dealer or drover	868	838	96.5
47	Has had inflammatory rheumatism once	3,547	3,442	97.0
24	Contractor	1,577	1,537	97.5
62	Weight B, neither parent noted as dead below 70	2,345	2,312	98.6
73	Height above six feet three inches	268	267	99.6
33	Railway passenger conductor	207	207	100.0
74	Height below five feet	93	93	100.0
71	Weight D, parent dead below 70 of any lung disease	1,864	1,869	100.3
46	Has had asthma	1,013	1,028	101.5
12	Physician	5,539	5,643	101.9
69	Weight C, both parents dead below 60	11,921	12,260	102.8
4	Born in Germany	19,314	20,512	106.2

List of Seventy-six Classes.

Arranged in Order of Committee's Percentages of Actual to Expected Death.

Class	Description	Committee's Expected Deaths	Actual Deaths	Percentage
32	Distiller or employee	192	205	106.8
53	Has had renal colic, calculus or gravel	462	495	107.1
52	Has had hepatic colic, or gall-stone	387	416	107.5
55	Has had blood spitting	756	814	107.7
1	Amount $20,000 or over on same application	1,860	2,006	107.8
48	Has had inflammatory rheumatism oftener than once	533	575	107.9
7	Colored	224	242	108.0
42	Seaman or fisherman	178	193	108.4
11	In city fire department	56	63	112.5
17	Glass worker	95	107	112.6
3	Granted on plan not applied for	1,501	1,704	113.5
8	Officer in army. Exclude issued after 1896	219	255	116.4
10	Marshal, sheriff, police, constable, etc	648	757	116.8
59	Weight A, neither parent noted as dead below 70	899	1,060	117.9
29	Wine or liquor seller, abstainer	344	411	119.5
15	Steel grinder	30	36	120.0
63	Weight B, one parent, or both dead below 70	4,549	5,465	120.1
28	Hotel keeper, not attending bar	972	1,188	122.2
23	Laborer	959	1,188	123.9
5	Born in Ireland	5,855	7,304	124.7
45	Has taken cure for, or has reformed from intemperance	238	299	125.6
37	Railway locomotive engineer	173	220	127.2
2	Amount applied for reduced by company	513	655	127.7
56	Has had hip-joint disease	104	133	127.9
25	Theatrical occupation	78	101	129.5
30	Wine or liquor seller, non-abstainer	1,300	1,704	131.1
64	Weight B, girth of abdomen greater than chest expanded	403	536	133.0
50	Has had syphilis	298	397	133.2
39	Officer of ocean steam vessel	99	133	134.3
31	Brewer or employee	588	796	135.4
9	Officer in navy	114	156	136.8
60	Weight A, one parent, or both, dead below 70	1,579	2,324	147.2
34	Railway passenger trainman. Exclude prior to 1890	16	24	150.0
61	Weight A, girth of abdomen greater than chest expanded	276	419	151.8
40	Officer of sailing vessel on Ocean or Great Lakes	266	408	153.4
49	Has had gout	109	168	154.1
38	Railway locomotive fireman	38	61	160.5
	Totals, including percentage ratio of total actual to total expected deaths	140,854	140,622	99.8

What first strikes the eye, on looking at this list, is the great number of classes which show a mortality better than the assumed standard. Out of the seventy-six classes no less than thirty-two present this favorable appearance, and they embrace 65,992 deaths out of the total of 140,622. Of these thirty-two, twenty-two embracing 37,189 deaths show a mortality less than ninety per cent. of the standard, and of the twenty-two, four embracing 1,443 deaths, show a mortality less than eighty per cent. of the standard.

It is noteworthy, also, that the aggregate deaths of the entire seventy-six classes are slightly less than the aggregate number expected according to the assumed standard. The inference is that the standard assumed is on the whole the equivalent of such mortality tables as the aggregated data of these seventy-six classes would have produced. It would not seem, *a priori*, a natural assumption, that the combined data of seventy-six classes, selected as these were, would give the same results as the entire business of the companies would have given if attainable. On the contrary, it would seem to be a safe proposition that since everyone of these seventy-six classes was supposed in advance to represent a blemish, it must follow that if the data of a large number of the best of the classes are combined, the tables deduced from such data could not, to any considerable extent, show a lighter mortality than would the ideal tables which would have been brought out from the entire business of the companies, good, bad and indifferent. To put the statement in another form, it is not likely that any large section of the impaired or classified business would show a lighter mortality than the total "accepted" business of the various companies. With this idea I have taken the first twenty-seven classes of the above table, those which show less than ninety-five per cent. of the standard mortality, and have combined the exposures and deaths of these twenty-seven classes by insurance years and ages of exposure. The total number of deaths contained in this body of experience is 57,596, being over forty per cent. of the entire number of deaths in the seventy-six classes.

The first question undertaken to be answered in connection with this mass of data was—How many distinct mortality tables for successive periods of duration of the insurances are necessary in order to present a true picture of the facts? In other words, at

what rate and how soon does the selection wear off? To answer this question the expected mortality according to the American Table has been calculated upon the exposures for each policy year separately, from the 1st to the 15th, then for the 16th to the 20th combined, and for the 21st to the 30th combined, and then the total number of the tabular deaths so found for each insurance year or group of years has been compared with the total number of actual deaths. The results of this preliminary comparison are given in the table below.

TWENTY-SEVEN CLASSES.

ACTUAL DEATHS BY INSURANCE YEARS, COMPARED WITH EXPECTED ACCORDING TO AMERICAN TABLE.

Insurance Year	Expected	Actual	Percentage of Actual to Expected
1	10,968	5,220	47.6
2	7,838	4,902	62.5
3	6,734	4,597	68.3
4	5,791	4,128	71.3
5	5,178	3,892	75.2
6	4,640	3,512	75.7
7	4,187	3,104	74.1
8	3,811	2,866	75.2
9	3,459	2,637	76.2
10	3,152	2,571	81.6
11	2,564	2,207	86.1
12	2,393	1,974	82.5
13	2,165	1,816	83.9
14	1,973	1,676	84.9
15	1,790	1,548	86.5
5th to 9th incl.	21,275	16,011	75.3
10th to 15th "	14,037	11,792	84.0
16th to 20th "	6,342	5,796	91.4
21st to 30th "	5,806	5,150	88.7

If we may assume that the percentages in the foregoing tables approximately represent the true movement we should have to conclude that the normal mortality relatively to tabular goes up by steps for four years only, then remains upon a level from the fifth to the ninth years inclusive, and then takes another upward step

and remains upon the level so reached from the tenth year to the fifteenth year inclusive, then takes one more upward step, finding its permanent level from the sixteenth year on.

I should interpret these figures as giving a fair picture of the normal Endowment mortality of the combined companies—the deterioration at the tenth year being due to the disappearance of the ten-year Endowments (the high death rate in the tenth year itself being presumably an accident), and the further deterioration at the fifteenth year being due to the disappearance of the fifteen-year Endowments.

The foregoing material suggests seven distinct mortality tables, representing successive stages in the lifetime of the insurances, as follows:

1st insurance year.
2d insurance year.
3d insurance year.
4th insurance year.
5th to 9th insurance years.
10th to 15th insurance years.
16th to 30th insurance years.

Mr. Arthur Hunter has undertaken for me the construction of these select tables, and furnishes the following statement:

"After the data were collected the ungraduated rate of mortality was obtained. This was then graduated by means of Mr. G. F. Hardy's method of using Woolhouse's formula. This method has not received the attention it deserves, as it appears only in the form of an actuarial note in Vol. XXXII of the *Institute Journal.* The method is so simple that it does not require a high grade of help to carry it out. The rule is as follows:

"Sum three times successively in groups of five, subtract three times the central second differences of the result so obtained, divide by 125, and the result is the adjusted value of u_o."

As we started with age fifteen the first q_x which was obtained by the above process of graduation is that for age twenty-three. The results of the graduation are given in the table below. The rate of mortality has been given by attained age in order that a comparison of the mortality for the same attained age at successive policy years can be readily instituted. The above system of

graduation does not smooth out all the irregularities, but we have refrained from carrying the work further because we thought it advisable not to eliminate irregularities which might be characteristic. It will be noticed that the mortality decreases with advancing years at the young ages. The same peculiarity may be noticed in the select tables prepared by Dr. Sprague from the H^M Experience. In those tables the mortality for the first calendar year decreases from age twenty to age twenty-seven, and does not exceed the mortality at age twenty until age thirty-six. Also, in the second, third and fourth calendar years the same condition is observed, although it does not cover so many ages. Even Dr. Sprague's rate of mortality for the fifth and subsequent calendar years is higher at age twenty-four than at any subsequent age until age thirty-eight is reached. In the first policy year in our experiment there is a steady increase in the rate of mortality from the earliest years, but in the second to the ninth years inclusive the same conditions are seen as in Dr. Sprague's tables although not to the same marked extent. The grouping of the data for the tenth to the fifteenth, and from the sixteenth to the thirtieth policy years shows an increasing mortality for each successive age, with slight exceptions.

Rates of Mortality,—Twenty-seven Classes.

Attained Age	Policy Years.						
	1st	2d	3d	4th	5th to 9th inclu.	10th to 15th inclu.	16th to 30th inclu.
23	.00330						
24	.00339	.00550					
25	.00345	.00530	.00606				
26	.00343	.00504	.00598	.00542			
27	.00347	.00483	.00557	.00538	.00607		
28	.00352	.00474	.00527	.00540	.00593		
29	.00351	.00470	.00513	.00540	.00593		
30	.00352	.00471	.00495	.00538	.00592		
31	.00358	.00486	.00500	.00535	.00593		
32	.00360	.00496	.00519	.00549	.00599	.00615	
33	.00362	.00502	.00544	.00555	.00605	.00659	
34	.00371	.00504	.00563	.00573	.00611	.00680	
35	.00387	.00501	.00591	.00588	.00620	.00686	
36	.00399	.00500	.00606	.00602	.00628	.00701	
37	.00415	.00510	.00619	.00608	.00639	.00687	
38	.00436	.00520	.00625	.00618	.00649	.00684	.00704
39	.00456	.00538	.00639	.00632	.00666	.00690	.00738
40	.00467	.00558	.00637	.00652	.00679	.00700	.00779
41	.00486	.00580	.00644	.00680	.00698	.00719	.00814
42	.00505	.00604	.00643	.00714	.00724	.00750	.00795
43	.00520	.00638	.00660	.00756	.00754	.00786	.00845
44	.00529	.00668	.00664	.00794	.00779	.00826	.00885
45	.00549	.00715	.00697	.00842	.00817	.00876	.00918
46	.00565	.00758	.00734	.00898	.00861	.00924	.00959
47	.00583	.00805	.00802	.00954	.00901	.00976	.01011
48	.00621	.00831	.00864	.01027	.00950	.01033	.01036
49	.00684	.00888	.00959	.01101	.01011	.01102	.01070
50	.00746	.00934	.01031	.01176	.01079	.01164	.01136
51	.00827	.00999	.01124	.01227	.01152	.01234	.01207
52	.00929	.01080	.01178	.01288	.01238	.01320	.01313
53	.01031	.01219	.01261	.01332	.01322	.01418	.01446

RATE OF MORTALITY FOR TWENTY-SEVEN CLASSES—*Concluded.*

ATTAINED AGE	POLICY YEARS.						
	1st	2d	3d	4th	5th to 9th inclu.	10th to 15th inclu.	16th to 30th inclu.
54	.01120	.01324	.01353	.01393	.01419	.01520	.01600
55	.01251	.01457	.01480	.01458	.01524	.01646	.01757
56	.01359	.01571	.01581	.01553	.01625	.01782	.01936
57	.01448	.01653	.01778	.01673	.01757	.01922	.02090
58	.01493	.01751	.01899	.01771	.01908	.02054	.02236
59	.01538	.01826	.01976	.01870	.02055	.02198	.02392
60	.01544	.01948	.02136	.01936	.02202	.02345	.02570
61	.01631	.02065	.02344	.02049	.02429	.02530	.02739
62	.01748	.02332	.02519	.02022	.02629	.02730	.02955
63	.01919	.02489	.02790	.02204	.02879	.02979	.03224
64	.....	.02610	.03155	.02443	.03178	.03251	.03489
65	.....	.....	.03155	.02953	.03470	.03591	.03748
66	.....	.....	.....	.03453	.03661	.03908	.04072
67	.....	.....	.....	.....	.04031	.04258	.04432
68	.....	.....	.....	.....	.04376	.04648	.04780
69	.....	.....	.....	.....	.04609	.04997	.05158
70	.....	.....	.....	.....	.05131	.05386	.05627
71	.....	.....	.....	.....	.05621	.05927	.06057
72	.....	.....	.....	.....	.....	.06563	.06475
73	.....	.....	.....	.....	.....	.07165	.06984
74	.....	.....	.....	.....	.....	.08002	.07578
75	.....	.....	.....	.....	.....	.08503	.08127
76	.....	.....	.....	.....	.....	.09219	.08843
77	.....	.....	.....	.....	.....	.....	.09662
78	.....	.....	.....	.....	.....	.....	.10319
79	.....	.....	.....	.....	.....	.....	.11142
80	.....	.....	.....	.....	.....	.....	.12294
81	.....	.....	.....	.....	.....	.....	.13988
82	.....	.....	.....	.....	.....	.....	.15039
83	.....	.....	.....	.....	.....	.....	.16607
84	.....	.....	.....	.....	.....	.....	.18440

The ultimate mortality rate of the twenty-seven classes, derived from all the exposures and deaths after the 15th year of insurance, has been graduated according to the Makeham formula, and the result is presented below, compared with the rate assumed by the Committee on the Specialized Investigation as ultimate standard.

Comparison of Ultimate Mortality Rates.

Attained Age	Twenty-seven Classes	Committee's ultimate Standard	Attained Age	Twenty-seven Classes	Committee's ultimate Standard
38	.00666	.00903	65	.03732	.03680
39	.00696	.00918	66	.04036	.03999
40	.00728	.00938	67	.04366	.04343
41	.00763	.00960	68	.04725	.04719
42	.00802	.00985	69	.05115	.05124
43	.00844	.01010	70	.05539	.05564
44	.00890	.01039	71	.06000	.06040
45	.00940	.01076	72	.06500	.06567
46	.00995	.01111	73	.07043	.07127
47	.01054	.01153	74	.07631	.07742
48	.01119	.01200	75	.08269	.08400
49	.01190	.01248	76	.08960	.09107
50	.01267	.01301	77	.09708	.09862
51	.01351	.01364	78	.10518	.10689
52	.01443	.01443	79	.11393	.11552
53	.01543	.01533	80	.12338	.12493
54	.01652	.01630	81	.13358	.13470
55	.01771	.01736	82	.14457	.14523
56	.01900	.01852	83	.15641	.15643
57	.02042	.01979	84	.16913	.16794
58	.02195	.02118	85	.18281	.18018
59	.02363	.02268	86	.19747	.19338
60	.02546	.02434	87	.21317	.20655
61	.02744	.02615	88	.22995	.22080
62	.02961	.02850	89	.24785	.23565
63	.03196	.03108	90	.26691	.25073
64	.03453	.03388	91	.28716	.26562

The two rates practically coincide, as will be seen, at and after the age of 51 ; but at the ages under forty-five the rate of the twenty-seven classes is materially lower than the other.

Taking the mortality rates of the twenty-seven classes for the successive insurance years as a standard of normal insured lives, it would be interesting and perhaps instructive to calculate the expected mortality by these rates, in the several classes of the Specialized Investigation, and to find the ratios of the actual to such expected mortality, and then to compare such ratios with those produced by the Committee. This has been done with three typical classes, and the results are given below.

CLASS 4—BORN IN GERMANY.

YEAR OF INSURANCE	Exposures (1)	Expected Mortality		Actual Mortality (4)	Ratios of Actual to Expected		Excess of (6) over (5)
		Committee (2)	27 Classes (3)		Committee (5)	27 Classes (6)	
1st	252,769	1,285	1,259	1,524	118.6%	121.0%	2.4%
2d	183,978	1,311	1,240	1,526	116.4	123.1	6.7
3d	155,814	1,374	1,189	1,435	104.4	120.7	16.3
4th	131,200	1,336	1,090	1,275	95.4	117.0	21.6
5th to 9th inclusive	449,252	5,477	4,413	5,285	96.5	119.8	23.3
10th to 15th inclusive	266,599	4,163	3,747	4,399	105.7	117.4	11.7
16th to 30th inclusive	176,762	4,368	4,337	5,068	116.0	116.9	.9
Total	1,616,374	19,314	17,275	20,512	106.2	118.7	12.5

Class 12—Physicians.

Year of Insurance	Exposures (1)	Expected Mortality		Actual Mortality (4)	Ratios of Actual to Expected		Excess of (6) over (5)
		Committee (2)	27 Classes (3)		Committee (5)	27 Classes (6)	
1st	98,683	463	442	512	110.6	115.8	5.2
2d	67,314	441	407	493	111.8	121.1	9.3
3d	56,290	455	384	456	100.2	118.8	18.6
4th	47,468	441	349	402	91.2	115.2	24.0
5th to 9th inclusive	159,759	1,752	1,369	1,614	92.1	117.9	25.8
10th to 15th inclusive	81,882	1,135	996	1,210	106.6	121.5	14.9
16th to 30th inclusive	39,278	852	836	956	112.2	114.4	2.2
Total	550,674	5,539	4,783	5,643	101.9	118.0	16.1

Class 31—Brewers or Employees.

1st	8,177	40	39	57	142.5	146.2	3.7
2d	6,356	43	41	57	132.6	139.0	6.4
3d	5,474	46	40	54	117.4	135.0	17.6
4th	4,634	45	36	46	102.2	127.8	25.6
5th to 9th inclusive	15,826	182	145	245	134.6	169.0	34.4
10th to 15th inclusive	8,764	127	112	193	152.0	172.3	20.3
16th to 30th inclusive	4,530	105	103	144	137.1	139.8	2.7
Total	53,761	588	516	796	135.4	154.3	18.9

The practical bearing of this experiment is to strengthen the emphatic warning of the Committee expressed in the following paragraph of the Introduction to the volume:

"One necessary warning cannot be expressed too strongly. It must not be forgotten that the facts herein given relate to the respective classes of risks among lives selected for insurance, and do not relate to the same classes among the general population. For example, it is not conceivable that, among the general population, those who have had at least one parent dying from consumption are above the average of the others in vitality. If this is found to be the case as regards that particular class of insured lives it indicates only that such persons of that class as have actually been accepted for insurance have been selected so carefully that, on the whole, those only have been accepted who are peculiarly good representatives of the class. If, on the other hand, the results appear only moderately bad upon a class of risks heretofore accepted with great circumspection, it is to be inferred that, had such circumspection not been exercised, the results would have been still worse. This warning must be borne in mind as applying and intended to apply, to each one of the classes under consideration."

It follows that if risks of any of the classes are taken wholesale, they may be expected to show a considerably higher mortality than that brought out in the investigation.

A further caution is also called for: it should be borne in mind that the investigation deals only with lives insured, but that the financial effect of insuring any of these classes will depend on the experience in respect to amounts insured, which, as commonly observed, is apt to be worse than the experience on lives, in groups in any way below the standard.

Frequency Curves and Moments.

ROBERT HENDERSON.

So much has recently been said on the subject of the use of frequency curves in all sorts of statistical work that it seems desirable that a concise statement of their general theory should be readily available for the actuarial student.

Perhaps for our purposes it would be more accurate to speak of a frequency function, because the curve only enters into the question as a convenient graphical representation of the function, and all the computations are made by the use of the algebraic function without any necessary reference to the geometrical curve. It has, however, become a well established practice to speak of the frequency curve, and no material advantage would be gained by departing from customary usage in the matter.

If a variable may have any value within certain limits, the curve, in which the ordinate y is proportional to the chance of the variable having the value x represented by the abscissa, is called the curve of frequency. For example, let the variable be the length of a human life which may have any value from zero to the limits of the mortality table. The chance of the duration falling between x *and* $x + dx$ is evidently $(l_x - l_{x+dx})/l_0$ or $l_x \mu_x \, dx/l_0$ so that the equation of the frequency curve in this case is $y = l_x \mu_x$.

Similarly, if two points be taken at random on a straight line, the curve of frequency of the distance of the furthest of the two from a specified end is represented by the straight line $y = x$ between the limits o and a where a is the length of the line.

Where in any two cases the variables are comparable quantities, such as the heights of individuals in two different nations or the durations of life in two different groups of individuals, it is convenient to have some short method of comparison between the two; some coefficient or factor which will indicate whether one curve or the other falls, on the whole, on the higher values of the variable, and if so by how much.

The quantity most frequently used for this purpose is the *mean* value of the variable obtained by multiplying each value by its probability and summing. In other words, if m_1 be the mean value and a and b the limits of the curve, we have $m_1 = \frac{\int_a^b y\,x\,dx}{\int_a^b y\,dx}$

It is evident from the above method that if in one curve the value of m_1 is greater than in another, the former falls on the average on higher values of the variable. It is also evident that represented geometrically, m_1 is the abscissa of the center of position of the area included between the curve and the base.

Other functions which have been used for this purpose are the *median* and the *mode*. The median is that value of the variable which it is as likely as not to exceed. Its value h is determined from the equation $\int_a^h y\,dx = \int_h^b y\,dx$ where a is the lower and b the upper limit of the value of x. The mode is the value of the variable whose probability is the greatest. For instance: in the case where the curve is represented by the equation $y = f(t) = l_{x+t}\,\mu_{x+t}$, t being the variable, we see that the mean value of t is the complete expectation of life at age x, that the median value is what is known as the *vie probable* or equation of life, and that the mode corresponds to the most probable after lifetime.

When the mean value of the variable has been determined, the next question is, "How closely do the values of the variable cluster about this mean value or how widely dispersed are they?" Several methods might be proposed of measuring the degree of dispersion, but the most natural one in connection with the mean is the mean square of the departure designated by μ_2 when the departure is measured from the mean.

The mean square of the departure from any given value is known as the second moment about that value. The value of the second moment m_2 about any given origin can be readily expressed in terms of m_1, the mean value of the variable, and μ_2 as follows:

Designating the operation of taking the mean value by writing M in front of the expression, we have

$$\mu_2 = \mathrm{M}\,(x - m_1)^2 = \mathrm{M}\,x^2 - 2\,m_1\,\mathrm{M}x + m_1^2 = m_2 - 2\,m_1^2 + m_1^2 = m_2 - m_1^2$$

or $m_2 = \mu_2 + m_1^2$.

It is thus evident that the second moment about any other value of the variable is greater than that about the mean value. In other words, taking the mean value as point of reference makes

the second moment a minimum. It thus appears that the second moment has a natural connection with the mean value of the variable. The mean absolute departure, departure in either direction being considered positive, has a similar connection with the median.

Other measures in terms of μ_2 are sometimes substituted for it in order to express the dispersion as a linear magnitude. The measures most frequently so used in connection with frequency curves in general are the *standard deviation* and the *modulus*. The standard deviation, commonly denoted by σ, is a quantity whose square is equal to the mean square of departure. In other words, $\sigma^2 = \mu_2$. The modulus, sometimes denoted by c, is determined by the equation $c^2 = 2\,\mu_2$, and the name is derived by analogy from the normal exponential frequency curve whose equation is $y = ke^{-\frac{x^2}{c^2}}$ in the case of which curve $\mu_2 = \frac{c^2}{2}$.

Having determined the mean value of the variable and the degree of dispersion from that mean value, the next question is whether the various possible values of the variable are dispersed symmetrically about the mean value or whether the curve is heaped up on one side and drawn out on the other. And as the first moment necessarily vanishes and the second moment can give us no information on the subject because departures in both directions enter into it positively, we are forced to look to the third moment, or the mean value of the cube of the departure, for a criterion. It is evident that if the curve is symmetrical, each positive departure will be balanced by a corresponding negative one, and so the mean value of the cube will vanish. It is thus evident that a value of the third moment, other than zero, is an indication of a lack of symmetry. Denoting by μ_3 the third moment about the mean and by m_3 the corresponding moment about any other point taken as origin, we have $\mu_3 = \mathrm{M}\,(x - m_1)^3$

$$= \mathrm{M}\,x^3 - 3\,m_1\,\mathrm{M}\,x^2 + 3\,m_1^2\,\mathrm{M}\,x - m_1^3 = m_3 - 3\,m_1\,m_2 + 2\,m_1^3.$$

$$= m_3 - 3\,m_1\,\mu_2 - m_1^3.$$

$$\text{or } m_3 = \mu_3 + 3\,m_1\,\mu_2 + m_1^3.$$

Of course, the curve is not necessarily absolutely symmetrical if μ_3 vanishes, but any marked lack of symmetry would likely show itself in the value of μ_3. The value of μ_3 is usually taken as a measure of the skewness or lack of symmetry, being divided

by c^3 in order that the measure may be always numerical, and the quotient being designated by j so that we have $j = \mu_3/c^3$. Another function entering into the theory of curves of frequency is the quotient of the square of the third moment by the cube of the second moment and denoted by β_1, so that we have $\beta_1 = \mu_3^2/\mu_2^3$. But we have $j^2 = \mu_3^2/c^6 = \mu_3^2/8\mu_2^3$ so that $\beta_1 = 8\, j^2$.

Similarly, further information in relation to the curve can be secured by determining the moments of higher order, but we shall in this investigation only take into account the fourth moment μ_4 or the mean value of the fourth power of the departure from the mean, and the quantity β_2 determined from the equation $\beta_2 = \mu_4/\mu_2^2$.

The value of m_4, the fourth moment about any other point as origin, may be readily seen to be connected with that of μ_4 by the following relation

$$\mu_4 = m_4 - 4\, m_1\, m_3 + 6 m_1^2\, m_2 - 3\, m_1^4 \text{ or}$$
$$m_4 = \mu_4 + 4\, m_1\, \mu_3 + 6\, m_1^2\, \mu_2 + m_1^4.$$

In practical cases it frequently occurs that the form of the curve of frequency cannot be determined exactly, but must be approximated to by observation of a limited number of cases. It is therefore of interest to investigate the method of determining approximately the values of the constants above defined, in cases where the number of observations is limited.

The similarity between the average of a finite number of cases and the mean value of an indefinitely large number, including the fact that the latter is the limit of the former when the number of cases is indefinitely increased, indicates the average as an approximate measure of the mean value. As each of the individual cases is, however, liable to variation, it is evident that the average is also so liable, and in fact has a curve of frequency of its own and the constants of this curve of frequency can be determined in terms of those of the primary curve. It is evident that the mean value of the average of n variable quantities is equal to the average of the n mean values of those quantities, and it follows that, if those n quantities follow the same law of distribution and have the same mean value, such mean value is also that of the average of the n quantities, so that the mean value in the derived curve is the same as in the primary curve. With regard to the

other moments, we have, taking the mean as origin and designating moments in the derived curve by accented letters.

$$\mu_2' = \mathrm{M}\left(\frac{x_1 + x_2 + \ldots\ldots + x_n}{n}\right)^2 = \mathrm{M}\ \frac{x_1^2 + x_2^2 + \ldots. x_n^2}{n^2}$$

$$+ \mathrm{M}\ \frac{2\,x_1\,x_2 + 2\,x_1\,x_3 + \ldots\ldots}{n^2} =$$

$$\frac{n}{n^2}\,\mathrm{M}\,x_1^2 + \frac{n\,(n-1)}{n^2}\,\mathrm{M}\,x_1\,x_2 = \frac{1}{n}\,\mu_2$$

$$\mu_3' = \mathrm{M}\left(\frac{x_1 + x_2 + \cdots x_n}{n}\right)^3 = \frac{1}{n^3}\,\mathrm{M}\,(x_1^3 + x_2^3 + \cdot\cdot x_n^3) +$$

$$\frac{1}{n^3}\,\mathrm{M}\,(3\,\Sigma\,x_1^2\,x_2 + 6\,\Sigma\,x_1\,x_2\,x_3) = \frac{1}{n^2}\,\mu_3$$

$$\mu_4' = \mathrm{M}\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)^4 = \frac{1}{n^4}\,\mathrm{M}\,\{(x_1^4 + x_2^4 + \cdots x_n^4) +$$

$$4\,\Sigma\,x_1^3\,x_2 + 6\,\Sigma\,x_1^2\,x_2^2 + 12\,\Sigma\,x_1^2\,x_2\,x_3 + 24\,\Sigma\,x_1\,x_2\,x_3\,x_4\}\ .$$

$$= \frac{1}{n^4}\,\{n\,\mu_4 + 3\,n\,(n-1)\,\mu_2^2\} = \frac{1}{n^3}\,\mu_4 + \frac{3\,(n-1)}{n^3}\,\mu_2^2$$

The above reductions follow from the principle that the mean value of the product of two independent variables is equal to the product of their respective mean values, and from the fact that the mean value of the first powers of the variables is zero. It is evident from the above that in relation to the other constants above mentioned we have $\sigma' = \frac{1}{\sqrt{n}}\,\sigma$; $c' = \frac{1}{\sqrt{n}}\,c$; $\beta_1' = \frac{\mu_3^2}{n^4} \div \frac{\mu_2^3}{n^3} = \frac{1}{n}\,\beta_1$; $j' = \frac{1}{\sqrt{n}}\,j$; and $\beta_2' = \frac{\mu_4 + 3\,(n-1)\,\mu_2^2}{n^3} \div \frac{\mu_2^2}{n^2}$

$$= \frac{1}{n}\,\{\beta_2 + 3\,(n-1)\} = 3 + \frac{1}{n}\,(\beta_2 - 3).$$

From these equations it is evident that as we take the average of a larger and larger number of cases, the values of β_1' and j' tend to vanish, that is, the curve becomes more and more symmetrical, and the value of β_2' tends to approach the value 3, which will be seen later to be the value of this function in the normal symmetrical exponential curve of frequency.

The second and higher moments, as derived from a finite number of observations, are subject to the additional source of error that, besides the variability of the individual values, the true position of the mean is not known and we are forced to use

the average of the known values as a substitute. The second moment, as derived from the observations, will be $\frac{1}{n}\Sigma\left(x_1-\frac{\Sigma x_1}{n}\right)^2$ $=\frac{1}{n}\left\{\Sigma x_1^2-2\Sigma x_1\frac{\Sigma x_1}{n}+n\left(\frac{\Sigma x_1}{n}\right)^2\right\}=\frac{\Sigma x_1^2}{n}-\frac{(\Sigma x_1)^2}{n^2}=$ $\left(\frac{1}{n}-\frac{1}{n^2}\right)\Sigma x_1^2-\frac{2}{n^2}\Sigma x_1 x_2$.

The mean value of this is evidently $\frac{n-1}{n}\mu_2$, so that the second moment, as derived from n observations, is on the average less than the true second moment by $\frac{1}{n}$ of itself. The third moment is similarly $\frac{1}{n}\Sigma\left(x_1-\frac{\Sigma x_1}{n}\right)^3=\frac{1}{n}\left\{\Sigma x_1^3-3\Sigma x_1^2\frac{\Sigma x_1}{n}+\right.$ $\left.3\Sigma x_1\left(\frac{\Sigma x_1}{n}\right)^2-n\left(\frac{\Sigma x_1}{n}\right)^3\right\}=\frac{1}{n}\Sigma x_1^3-\frac{3}{n^2}(\Sigma x_1^3+\Sigma x_1^2 x_2)$ $+\frac{2}{n^3}(\Sigma x_1^3+3\Sigma x_1^2 x_2+6\Sigma x_1 x_2 x_3)$.

The mean value of this is $\left(1-\frac{3}{n}+\frac{2}{n^2}\right)\mu_2=\frac{(n-1)(n-2)}{n^2}\mu_2$.

The fourth moment, as derived from the observations, is $\frac{1}{n}\Sigma\left(x_1-\frac{\Sigma x_1}{n}\right)^4=\frac{1}{n}\{\Sigma x_1^4-\frac{4}{n}\Sigma x_1^3\Sigma x_1+\frac{6}{n^2}\Sigma x_1^2(\Sigma x_1)^2$ $-\frac{3}{n^3}(\Sigma x_1)^4\}=\frac{1}{n}\Sigma x_1^4-\frac{4}{n^2}(\Sigma x_1^4+\Sigma x_1^3 x_2)+\frac{6}{n^3}(\Sigma x_1^4+$ $\Sigma x_1^2 x_2^2+2\Sigma x_1^3 x_2+2\Sigma x_1^2 x_2 x_3)-\frac{3}{n^4}(\Sigma x_1^4+4\Sigma x_1^3 x_2+$ $6\Sigma x_1^2 x_2^2+12\Sigma x_1^2 x_2 x_3+24\Sigma x_1 x_2 x_3 x_4)$.

The mean value of this is $\mu_4-\frac{4}{n}\mu_4+\frac{6}{n^2}\left(\mu_4+\frac{n-1}{2}\mu_2^2\right)-$ $\frac{3}{n^3}[\mu_4+3(n-1)\mu_2^2]=\mu_4\left(1-\frac{4}{n}+\frac{6}{n^2}-\frac{3}{n^3}\right)+\frac{3(n-1)}{n^2}$ $\mu_2^2\left(1-\frac{3}{n}\right)=\frac{(n-1)(n-2)}{n^2}\mu_4-\frac{(n-1)(n-3)}{n^3}(\mu_4-3\mu_2^2)$.

Where n, the number of observations, is large, this correction is, of course, practically negligible.

There are other cases where we are seeking, not the ideal ultimate moments which would result from an indefinitely large number of observations, but a concrete method of representing

the results of a finite number of observations, and, in such, the uncorrected moments, as derived from those observations, should be used.

A problem in connection with the calculation of these moments arises from the fact that where the number of observations is large, we are usually given, not the exact value of the variable resulting from each observation, but the total number of cases in which that value fell within each of a number of specified intervals in value, usually proceeding by equal differences, the problem being to determine the moments from the data so given. For example: in the case of a mortality experience, we are given the total number exposed to risk in each year of age and also the number of deaths in the same interval. In such cases it is convenient to work with a function representing the total number of cases greater than a given value, as the exact value of this function can be determined for the values of the variable corresponding to the points of division between the various intervals. If this function be denoted by $\mathrm{F}(x)$, then the equation of the curve of frequency of the variable will be $y = -\frac{d\mathrm{F}(x)}{dx}$. We have then generally $\int_a^b (x-a)^n\, y\, dx = -\int_a^b (x-a)^n\, d\,\mathrm{F}(x) = -[(x-a)^n\,\mathrm{F}(x)]_a^b + n\int_a^b (x-a)^{n-1}\,\mathrm{F}(x)\,dx$. But the first term on the right-hand side vanishes for both limits, if n be greater than zero, so that we have $\int_a^b (x-a)^n\, y\, dx = n\int_a^b (x-a)^{n-1}\,\mathrm{F}(x)\,dx$ where a and b are limits of value within which all the observed values of the variable lie.

But we have approximately $\int_a^b u_x\,dx = \Sigma_a^b\, u_x - \frac{1}{2}(u_a + u_b) + \frac{1}{12}\left(\frac{d\,u_a}{dx} - \frac{d\,u_b}{dx}\right) - \frac{1}{720}\left(\frac{d^3 u_a}{dx^3} - \frac{d^3 u_b}{dx^3}\right)$.

Calculating then the respective moments about $x = a$ we have, if we assume that the various derived functions of $\mathrm{F}(x)$ vanish for $x = a$ and $x = b$ or, in other words, that the curve of frequency has high contact with the curve at the limiting values,

$$\begin{aligned} m_1\,\mathrm{F}(a) &= \int_a^b (x-a)\, y\, dx = \int_a^b \mathrm{F}(x)\,dx = \Sigma_a^b\,\mathrm{F}(x) - \tfrac{1}{2}\,\mathrm{F}(a) \\ &= \Sigma_a^{b-1}\,(x-a+\tfrac{1}{2})\{\mathrm{F}(x)-\mathrm{F}(x+1)\}, \text{ since } \mathrm{F}(b) \text{ vanishes.} \\ m_2\,\mathrm{F}(a) &= 2\int_a^b (x-a)\,\mathrm{F}(x)\,dx = 2\,\Sigma_a^b\,(x-a)\,\mathrm{F}(x) + \tfrac{1}{6}\,\mathrm{F}(a) \\ &= \Sigma_a^{b-1}\,(x-a+\tfrac{1}{2})^2\,\{\mathrm{F}(x) - \mathrm{F}(x+1)\} - \tfrac{1}{12}\,\mathrm{F}(a). \\ m_3\,\mathrm{F}(a) &= 3\int_a^b (x-a)^2\,\mathrm{F}(x)\,dx = 3\,\Sigma_a^b\,(x-a)^2\,\mathrm{F}(x) \\ &= \Sigma_a^{b-1}\,\{(x-a+\tfrac{1}{2})^3 - \tfrac{1}{4}\,(x-a+\tfrac{1}{2})\}\{\mathrm{F}(x)-\mathrm{F}(x+1)\} \\ &= \Sigma_a^{b-1}(x-a+\tfrac{1}{2})^3\,\{\mathrm{F}(x) - \mathrm{F}(x+1)\} - \tfrac{1}{4}\,m_1\,\mathrm{F}(a). \end{aligned}$$

$$m_4 F(a) = 4\int_a^b (x-a)^3 F(x)\,dx = 4\,\Sigma_a^b (x-a)^3 F(x) - \tfrac{1}{30} F(a)$$
$$= \Sigma_a^{b-1} (x-a+\tfrac{1}{2})^4 \{F(x) - F(x+1)\}$$
$$- \tfrac{1}{2}\Sigma_a^{b-1} (x-a+\tfrac{1}{2})^2 \{F(x) - F(x+1)\} + \tfrac{7}{240} F(a).$$
$$= \Sigma_a^{b-1} (x-a+\tfrac{1}{2})^4 \{F(x) - F(x+1)\} - \tfrac{1}{2} m_2 F(a) - \tfrac{1}{80} F(a).$$

It is evident in the above equations that $F(x) - F(x+1)$ is the number of cases in which the value of the variable falls between x and $x+1$, and from the first equation we see that the mean value can be correctly determined by assuming all the cases in each interval to be concentrated at the middle point of the interval. If we designate then by accented letters the second and higher moments computed on that assumption, we have from the remaining equations

$$m_2 = m_2' - \tfrac{1}{12}$$
$$m_3 = m_3' - \tfrac{1}{4} m_1$$
$$m_4 = m_4' - \tfrac{1}{2} m_2' + \tfrac{7}{240}$$

Or, reducing to the mean value as point of reference

$$\mu_2 = m_2 - m_1^2 = m_2' - m_1^2 - \tfrac{1}{12} = \mu_2' - \tfrac{1}{12}$$
$$\mu_3 = m_3 - 3 m_1 m_2 + 2 m_1^3 = m_3' - \tfrac{1}{4} m_1 - 3 m_1 m_2' + \tfrac{1}{4} m_1 + 2 m_1^3 = \mu_3'$$
$$\mu_4 = m_4 - 4 m_1 m_3 + 6 m_1^2 m_2 - 3 m_1^4$$
$$= m_4' - \tfrac{1}{2} m_2' + \tfrac{7}{240} - 4 m_1 m_3' + m_1^2 + 6 m_1^2 m_2' - \tfrac{1}{2} m_1^2 - 3 m_1^4$$
$$= m_4' - 4 m_1 m_3' + 6 m_1^2 m_2' - 3 m_1^4 - \tfrac{1}{2} m_2' + \tfrac{1}{2} m_1^2 + \tfrac{7}{240}$$
$$= \mu_4' - \tfrac{1}{2}\mu_2' + \tfrac{7}{240} = \mu_4' - \tfrac{1}{2}\mu_2 - \tfrac{1}{80}$$

These formulas give the corrections to be applied to the calculated second and fourth moments in order to determine their true value and show that the third moment is obtained correctly without adjustment.

Hitherto we have dealt with general principles applicable to any curve of frequency, but there is a special family of such curves which have in recent years almost appropriated the title to themselves. I refer to the family of curves whose general differential equation takes the form, when the mean value is taken as the origin of x, of $\frac{1}{y}\frac{dy}{dx} = \frac{-(b+x)}{c+bx+ax^2}$. This family of curves has come into prominence partly because it includes as one of its members the curve of frequency, as given in the theory of inverse probability, of the probability of an event which has been observed to happen a specified number of times out of a given

number of trials, and as another member the well known exponential curve of error. But its principal claim to notice arises from the fact that a great number of the distributions occurring in nature appear to closely resemble some one of the types of curve included in this family.

It is evident from the form of the equation, that *log y* becomes, in general, infinite either positively or negatively along with x and also for values of x for which the denominator vanishes, so that these values of x will be the limits of the curve, y either vanishing or becoming infinite. We have, therefore, three essentially different cases according as the roots of the equation $a x^2 + b x + c = 0$ are real and of different signs, real and of the same sign, or complex. In the first case, the curve is limited in both directions, the two roots of the equation being the limiting values. In the second case, it is limited in one direction by the numerically least of the two roots, but extends to infinity in the other direction, while in the last case the curve is unlimited in both directions. In passing from the first to the second curve, we have a limiting case where one root is infinite and in passing from the second to the third, we have the case where the two roots are equal; in both of these cases the curve is limited in one direction and unlimited in the other. Of sufficient importance to be distinguished from these are the three cases where b vanishes, the cases being distinguished according as a/c is negative, positive or zero. The first of these is a particular case of the first type above mentioned, and the second of them is a particular case of the third type, while the last is a limiting case through which one passes into the other. In this last case, the equation takes the form

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log y = -\frac{x}{c}$$

Hence $\log y = \log y_o - \frac{x^2}{2c}$

or $y = y_o\, e^{-\frac{x^2}{2c}}$

This is the well known exponential law of error. We have thus, in all, eight distinct types of curve belonging to this family.

Independently, however, of the type of curve involved, we can, subject only to certain conditions, express the coefficients of the differential equation in terms of the various moments, and it will be seen by an examination of these conditions that they are

really only the conditions that the various moments involved should not become infinite.

From the fundamental equation we have

$$(b + x)\, y = -(c + b\, x + a\, x^2)\frac{d\, y}{d\, x}.$$

Suppose now l and m are the limiting values of the curve, these values being not necessarily finite.

Then $\int_l^m (b + x)\, x^{n-1}\, y\, dx$

$$= -\int_l^m (c\, x^{n-1} + b\, x^n + a\, x^{n+1})\frac{d\, y}{d\, x}\, d\, x$$

$$= -\left[(c\, x^{n-1} + b\, x^n + a\, x^{n+1})\, y\right]_l^m + \int_l^m \{(n-1)\, c\, x^{n-2} + n\, b\, x^{n-1} + (n+1)\, a\, x^n\}\, y\, dx$$

or, if $(c + b\, x + a\, x^2)\, x^{n-1}\, y$ vanishes for both limits,

$$b\, \mu_{n-1} + \mu_n = (n-1)\, c\, \mu_{n-2} + n\, b\, \mu_{n-1} + (n+1)\, a\, \mu_n$$

or $\{1 - (n+1)\, a\}\, \mu_n = (n-1)\, b\, \mu_{n-1} + (n-1)\, c\, \mu_{n-2}$

Putting then n equal in succession to 1, 2, 3 and 4, we get, since $\mu_o = 1$, $(1 - 2\, a)\, \mu_1 = 0$, as we should, since the mean value has been taken as the origin,

$$(1 - 3\, a)\, \mu_2 = c$$
$$(1 - 4\, a)\, \mu_3 = 2\, b\, \mu_2$$
$$(1 - 5\, a)\, \mu_4 = 3\, b\, \mu_3 + 3\, c\, \mu_2$$

Eliminating b and c from these equations, we have

$$2\,(1 - 5\, a)\, \mu_4\, \mu_2 - 3\,(1 - 4\, a)\, \mu_3^2 - 6\,(1 - 3\, a)\, \mu_2^3 = 0$$

or, dividing through by μ_2^3,

$$2\,(1 - 5\, a)\, \beta_2 - 3\,(1 - 4\, a)\, \beta_1 - 6\,(1 - 3\, a) = 0$$

whence $a = \dfrac{2\,\beta_2 - 3\,\beta_1 - 6}{10\,\beta_2 - 12\,\beta_1 - 18} = \dfrac{2\,(\beta_2 + 3) - 3\,(\beta_1 + 4)}{10\,(\beta_2 + 3) - 12\,(\beta_1 + 4)} = \dfrac{2 - 3\,\gamma}{10 - 12\,\gamma}$

if we put γ for $\dfrac{\beta_1 + 4}{\beta_2 + 3}$.

Substituting then in the first two equations, we have

$$b = \frac{1 - 4\, a}{2}\,\frac{\mu_3}{\mu_2} = \frac{1}{10 - 12\,\gamma}\,\frac{\mu_3}{\mu_2}$$

$$c = (1 - 3\, a)\, \mu_2 = \frac{4 - 3\,\gamma}{10 - 12\,\gamma}\, \mu_2$$

The criterion of the nature of the roots of the equation $a\, x^2 + b\, x + c = 0$ is $k_2 = \dfrac{b^2}{4\, a\, c} = \dfrac{1}{4\,(2 - 3\,\gamma)(4 - 3\,\gamma)}\,\dfrac{\mu_3^2}{\mu_2^3} = \dfrac{\beta_1}{4\,(2 - 3\,\gamma)(4 - 3\,\gamma)}$

If this is negative, the roots are real and of different signs; if it is positive and greater than unity, they are real and of the same sign; while if it is positive and less than unity, they are

complex ; if k_2 is infinite, one root of the equation is infinite ; while if it is equal to unity, the two roots are equal. The case where k_2 and consequently b vanishes requires a further criterion and we have seen that $\frac{a}{c}$ or $\frac{2-3\gamma}{4-3\gamma}\frac{1}{\mu_2}$ forms this criterion. Since μ_2 is essentially a positive quantity, this is negative if γ lies between $^2/_3$ and $^4/_3$, zero or infinite according as γ has one or the other of those values, and positive otherwise. It is evident from the form of the expression for k_2 that it is negative if $9(1-\gamma)^2$ is less than unity, positive and greater than unity if $9(1-\gamma)^2$ is greater than unity and less than $1+\frac{\beta_1}{4}$, and positive and less than unity if $9(1-\gamma)^2$ is greater than $1+\frac{\beta_1}{4}$; also that $\frac{a}{c}$ is negative if $9(1-\gamma)^2$ is less than unity and positive if it is greater, so that the value of γ relatively to that of β_1 forms a criterion of the type of curve which will give the required moments.

We have then the following eight types:

	Value of $9(1-\gamma)^2$	Value of β_1
I.....	$= 1$	$= 0$
II.....	$= 1$	> 0
III.....	< 1	> 0
IV.....	< 1	$= 0$
V.....	$> 1 < 1 + \frac{\beta_1}{4}$ ..	> 0
VI.....	$= 1 + \frac{\beta_1}{4}$	> 0
VII.....	$> 1 + \frac{\beta_1}{4}$	> 0
VIII.....	> 1	$= 0$

With the integration of this function and the consequent determination of the equation it is unnecessary to deal here, as it reduces readily to one of the fundamental forms taken up in all treatises on the Infinitesimal Calculus. I shall, therefore, content myself with appending a table, showing the equation of each type in its simplest form together with a statement of the mean value of x, of the values of μ_2, β_1, γ and k_2 and of the limitations to which the constants in the equation are subject in order that the conditions on page 39 may be satisfied. On examination it will be seen that these are also the conditions that the area of the curve

should be finite without k vanishing and that the values of the various moments up to the fourth inclusive should be finite (see page 37). I have assumed, in the case of the curves which are limited in one direction, that the skewness is positive and consequently that the direction which is limited is the negative one. The equation for curves of negative skewness can be obtained by putting $-x$ for x in the formula and changing the signs in the column giving the limits of the value of x and also in the mean value column.

Where it is desired to fit a curve, following any other general law, to a given set of observations, the general method would be similar to that above indicated for this particular family of curves. The values of the various moments of any selected point should be expressed in terms of the arbitrary constants, involved in the general form of the equation. A sufficient number of orders of moments must be used to give equations equal in number to the arbitrary constants involved. From these simultaneous equations the values of the arbitrary constants would then be determined. Of course, it is readily seen that an essential condition of the applicability of this method is the possibility of expressing the various moments as some known function of the arbitrary constants.

Types of Frequency Curves

Type	Equation $y =$	Limits of x		Mean m_1	μ_2	β_1	γ	κ_2	Limitations of Constants
		Lower	Upper						
I	$k\, e^{-\frac{x^2}{a^2}}$	$-\infty$	$+\infty$	0	$\frac{1}{2}\, a^2$	0	$2/3$	0	
II	$k\, x^{m-1}\, e^{-\frac{x}{a}}$	0	$+\infty$	$m\, a$	$m\, a^2$	$\frac{4}{m}$	$2/3$	∞	$m > 0;\ a > 0$
III	$(p + q = 1)$ $k\, (a-x)^{np-1}\, (a+x)^{nq-1}$	$-a$	$+a$	$(q-p)\, a$	$\frac{4\, p\, q}{n+1}\, a^2$	$\frac{4\,(n+1)\,(p-q)^2}{(n+2)^2\, p\, q}$	$\frac{2}{3} \cdot \frac{n+3}{n+2}$	$-\left(\frac{1}{4\, p\, q} - 1\right)$	$p > 0;\ q > 0;\ n > 0;\ a > 0$
IV	$k\, (a^2 - x^2)^{\frac{n}{2}-1}$	$-a$	$+a$	0	$\frac{a^2}{n+1}$	0	$\frac{2}{3} \cdot \frac{n+3}{n+2}$	0	$n > 0;\ a > 0$
V	$(q - p = 1)$ $k\, (x-a)^{np-1}\, (x+a)^{-(nq+1)}$	$+a$	$+\infty$	$(p+q)\, a$	$\frac{4\, p\, q}{n-1}\, a^2$	$\frac{4\,(n-1)\,(p+q)^2}{(n-2)^2\, p\, q}$	$\frac{2}{3} \cdot \frac{n-3}{n-2}$	$\frac{1}{4\, p\, q} + 1$	$p > 0;\ n > 3;\ a > 0$
VI	$k\, x^{-(n+2)}\, e^{-\frac{a}{x}}$	0	$+\infty$	$\frac{a}{n}$	$\frac{a^2}{n^2\,(n-1)}$	$\frac{16\,(n-1)}{(n-2)^2}$	$\frac{2}{3} \cdot \frac{n-3}{n-2}$	1	$n > 3;\ a > 0$
VII	$k(a^2+x^2)^{-(\frac{n}{2}+1)}\, e^{\nu \tan^{-1}\frac{x}{a}}$	$-\infty$	$+\infty$	$\frac{\nu\, a}{n}$	$\frac{n^2 + \nu^2}{n^2\,(n-1)}\, a^2$	$\frac{16\,(n-1)}{(n-2)^2} \cdot \frac{\nu^2}{n^2 + \nu^2}$	$\frac{2}{3} \cdot \frac{n-3}{n-2}$	$\frac{\nu^2}{n^2 + \nu^2}$	$n > 3$
VIII	$k\, (a^2 + x^2)^{-(\frac{n}{2}+1)}$	$-\infty$	$+\infty$	0	$\frac{a^2}{n-1}$	0	$\frac{2}{3} \cdot \frac{n-3}{n-2}$	0	$n > 3$

The Decline in the Birth Rate in New South Wales.

RICHARD TEECE.

It is generally accepted as a truism by political economists that an increasing marriage rate is the accompaniment of increasing prosperity and is a consequence of it; an increasing marriage rate ought to be attended by an increasing birth rate.

If these economic dogmas be true in a general sense their truth ought to be especially evident in a young country for the following reasons:

1. In a young country the proportion of the population which is young and able-bodied is usually larger than in an old one.

2. A young country being more sparsely settled than an old one there is greater room for expansion, and more vacant land available for settlement and at lower prices.

3. A greater proportion of the population being wage-earners, there is generally a higher standard of living.

Against this it is generally the case in a young country that the males are largely in excess of the females; there cannot be marriages without females and there ought not to be children without marriages.

The recent pamphlet of Mr. Coghlan on "The Decline in the Birth Rate" has caused somewhat of a shock to the State of New South Wales and a Royal Commission is now engaged inquiring into the causes which have led to this result. The case for the Commission is founded on the Table on page 4 of Mr. Coghlan's book giving "the number of legitimate births to

every 1,000 married women under the age of forty-five years." The table is as follows:

PERCENTAGE.

Year.	Rate.	Year.	Rate.	Year.	Rate.
1881	336.3	1888	328.8	1895	260.3
1882	330.5	1889	304.2	1896	240.4
1883	333.6	1890	304.7	1897	241.3
1884	346.2	1891	288.7	1898	229.6
1885	341.7	1892	286.4	1899	226.6
1886	338.9	1893	281.4	1900	227.1
1887	333.4	1894	267.1	1901	235.3

In England and Wales, at the census of 1901, the births among married women of ages fifteen to forty-five were 235 per 1,000, or just equal to those in New South Wales.

The rates for New South Wales run somewhat irregularly, but there can be no mistake about the tendency—the decrease has been about thirty per cent., or nearly one-third in twenty-one years.

It is worthy of note that the most marked decline followed the financial crisis of 1893 and accompanied the long drought which has just broken up.

Are these figures to be accepted as conclusive proof of the alleged fact? If so, what are the causes which have contributed to this result? With the two-fold view of helping to an answer to these questions and of affording to those interested an opportunity to study the subject I have compiled a number of Tables bearing on them; and these I now submit, together with such comments as their teaching appears to me to warrant. Table I is merely preliminary, and deals with male population at the three censuses of 1881, 1891 and 1901.

TABLE I.

Census Returns Relating to Male Population, 1881-1901.

Census.	Total Population (*a*)	Male Population (*b*)	Percentage of Male Population on Total Population.	Males, Ages 20-60 (*c*)	Percentage of Males (ages 20-60) on Total Population.	Percentage of Col. (*c*) on Col. (*b*).	Married Males, Ages 20-60.	Percentage of Married Males (ages 20-60) on Total Population.	Percentage of Married Males on Total Males of the same Ages.			
									20 and under 60	20 and under 30	30 and under 40	40 and under 60
1881	751,468	411,149	54.7	200,208	26.6	48.7	98,841	13.2	49.4	26.7	61.1	64.6
1891	1,132,234	612,562	54.1	308,322	27.2	50.3	150,772	13.3	48.9	25.6	60.8	65.8
1901	1,359,133	712,456	52.4	347,853	25.6	48.8	178,929	13.2	51.4	23.5	61.4	69.6

This table shows:

1st. That the ratios of the male population of all ages, and of that between ages twenty to sixty, to the total population has slightly decreased.

2d. That the ratio of married males aged twenty to sixty to the total population has remained practically constant.

Were we drawing a deduction from this second ratio alone, we should arrive at the conclusion that, as far as the result is influenced by this circumstance, the birth rate should have been constant; but when we come to distribute the ratios according to ages of married males, we find:

1st. That between ages twenty and thirty the ratio has sensibly declined.

2d. That between thirty and forty it has remained almost constant.

3d. That between forty and sixty it has sensibly increased.

In other words, the proportion of young married men has decreased, while that of the older ones has increased.

If, therefore we deduced our conclusions from these facts alone, we should look for a declining birth rate.

The complete figures for the full twenty-one years, from which those given in Table I. are condensed, will be found in Table II., page 46.

TABLE II.

NEW SOUTH WALES MALE POPULATION STATISTICS. COMPUTED FROM CENSUS RETURNS 1871, 1881, 1891 AND 1901.

Year. 1.	Total Population at Dec. 31st. 2.	Total Male Population at Dec. 31st. 3.	Percentage Column 3 on Column 2. 4.	Males, Ages 20 to 30. 5.	Percentage Column 5 on Column 3. 6.	Males, Ages 30 to 60. 7.	Percentage Column 7 on Column 3. 8.	Males, Ages 20 to 30, Married. 9.	Percentage Column 9 on Column 5. 10.	Males, Ages 30 to 60, Married. 11.	Percentage Column 11 on Column 7. 12.
1880	743,888	406,993	54.71	74,140	18.22	124,012	30.47	19,843	26.76	77,944	62.85
1881	774,994	423,549	54.65	77,569	18.31	129,311	30.53	20,667	26.64	81,337	62.90
1882	807,400	440,779	54.59	81,155	18.41	134,836	30.59	21,525	26.52	84,878	62.95
1883	841,161	458,709	54.53	84,909	18.51	140,598	30.65	22,419	26.40	88,573	62.99
1884	876,335	477,369	54.47	88,835	18.61	146,605	30.71	23,350	26.28	92,429	63.05
1885	912,979	496,788	54.41	92,944	18.71	152,869	30.77	24,319	26.17	96,453	63.10
			54.56		**18.47**		**30.63**		**26.45**		**62.98**
1886	951,155	516,996	54.35	97,242	18.81	159,401	30.83	25,329	26.05	100,652	63.14
1887	990,927	538,027	54.30	101,739	18.91	166,212	30.89	26,380	25.93	105,034	63.19
1888	1,032,363	559,914	54.24	106,444	19.01	173,314	30.95	27,476	25.81	109,607	63.24
1889	1,075,532	582,690	54.18	111,367	19.11	180,720	31.01	28,616	25.70	114,379	63.29
1890	1,120,513	606,395	54.12	116,516	19.21	188,442	31.08	29,805	25.58	119,359	63.34
			54.23		**19.02**		**30.96**		**25.80**		**63.25**
1891	1,141,831	616,014	53.95	116,754	18.95	192,055	31.18	29,647	25.39	122,107	63.58
1892	1,163,554	625,786	53.78	116,993	18.70	195,736	31.28	29,490	25.21	124,919	63.82
1893	1,185,691	635,713	53.62	117,232	18.44	199,489	31.38	29,333	25.02	127,795	64 06
1894	1,208,249	645,797	53.45	117,471	18.19	203,313	34.48	29,177	24.84	130,738	64.30
1895	1,231,236	656,041	53.28	117,711	17.94	207,211	31.59	29,023	24.66	133,748	64.55
			53.61		**18.44**		**31.38**		**25.02**		**64.07**
1896	1,254,660	666,448	53.12	117,951	17.70	211,183	31.69	28,869	24.48	136,828	64.79
1897	1,278,530	677,020	52.95	118,192	17.46	215,232	31.79	28,716	24.30	139,978	65.04
1898	1,302,854	687,760	52.79	118,434	17.22	219,358	31.89	28,563	24.12	143,201	65.28
1899	1,327,641	698,669	52.62	118,676	16.99	223,563	32.00	28,412	23.94	146,499	65.53
1900	1,352,896	709,755	52.46	118,919	16.75	227,850	32.10	28,261	23.76	149,872	65.78
			52.78		**17.22**		**31.90**		**24.12**		**65.29**

The percentages given in columns 4, 6, 8, 10 and 12 of Table II are shown in condensed form below:

Period.	Column 4.	Column 6.	Column 8.	Column 10.	Column 12.
1880-1885	54.56	18.47	30.63	26.45	62.98
1886-1890	54.23	19.02	30.96	25.80	63.25
1891-1895	53.61	18.44	31.38	25.02	64.07
1896-1900	52.78	17.22	31.90	24.12	65.29

This summary shows, as I have already pointed out, that while the proportion of males twenty to thirty who are married has decreased, the corresponding proportion of those thirty to sixty has increased. The natural effect of this would be a decline in the birth rate.

As a further preliminary to the investigation it may be stated that the marriages per thousand for the census years 1881, 1891, 1901, and the year 1902 on the total mean population of those years were as follows:

1881... 8.21 | 1891... 7.41 | 1901... 7.68 | 1902... 7.52

The mean age of women at marriage for the years 1897-1902 (the only ones for which the information is available) has been practically constant at twenty-four and one-half years. The corresponding age in England and Wales for a similar period was 26.2 years.

There is nothing in these figures to lead us to predicate any material decrease in the birth rate, and we must look elsewhere for an explanation.

The following Table gives the numbers of married women in various groups of ages at the periods indicated:

New South Wales.

Number of Women returned as "Married" in Age Groups

Ages.	According to the Census of			Ages.
	1881.	1891.	1901.	
under 15	8	—	—	under 15
15 and under 20	2,130	2,875	2,566	15 and under 20
20 " 25	14,967	20,043	19,672	20 " 25
25 " 30	18,638	32,041	32,618	25 " 30
30 " 35	17,563	29,505	34,608	30 " 35
35 " 40	16,117	22,584	33,108	35 " 40
40 " 45	22,101	18,018	26,900	40 " 45
45 " 50		14,549	18,761	45 " 50
50, etc.	16,310	26,033	38,086	50, etc.
Total.	107,834	165,648	206,319	Total.

The following Table summarizes the figures just given, while the results are graphically shown in the accompanying diagram:

New South Wales.

Women Returned as "Married."

Census of	1881.	1891.	1901.
Total Married	107,834	165,648	206,319
Number—			
Ages 15 and under 35	53,298	84,464	89,464
Ages 35 and over ...	54,536	81,184	116,855
Percentage of total—			
Ages 15 and under 35	49.5	51.0	43.4
Ages 35 and over ...	50.5	49.0	56.6

This Table shows that while the proportions of married women at ages up to thirty-four did not differ materially from those above that age at the census periods of 1881 and 1891, the figures for the census of 1901 show that the married women of

ages thirty-five and upwards exceed those below the age of thirty-five by 30.6 per cent. This change in the proportions has been coincident with the most rapid decline in the birth rate. I have already shown that the proportion of married men at the old ages has increased, while that of married men at the younger ages has decreased. We have therefore established the fact that both as regards men and women there are now proportionately much larger numbers at old ages and much smaller ones at young ages than was the case formerly, and that consequently a decreased birth rate is a natural result. Unfortunately, the figures do not afford us the means of determining the weight of this influence, and consequently we are unable to determine the extent to which the decrease is referable to this cause.

In Table III I present a series of figures from which almost any required deduction necessary to the intelligent discussion of the question can be drawn. For facility of comparison I have inserted the average percentages for one six-year period and for three five-year ones.

Column 4 shows that between the first and last periods the birth rate on the mean population has fallen about 26.2 per cent.; while column 6 shows that the corresponding rate on the mean female population has fallen about 28.8 per cent. Column 8, which is more to the point, shows that the fall in rate among the mean female population, ages fifteen to fifty (which may be taken as the extremes of the child-bearing period), has been about 32.8 per cent. These figures taken by themselves appear to afford conclusive proof that there has been a very serious decline in the birth rate; but the student of statistics knows that deductions from these figures may be absolutely unreliable.

We notice from column 13 that the percentage of the mean married female population on the mean total female population has remained constant throughout the entire period, and, omitting for the present the effect of any modifying influences, this would lead us to infer that the birth rate ought to have remained constant also.

We proceed, therefore, to deal with the marriage rate, and we see from column 15 that the marriage rate among females of the ages fifteen to fifty has decreased in the period under review by about 24.6 per cent. This decrease leads us naturally to look for a corresponding decrease in the birth rate.

TABLE III.

Table of various Mean Populations in New South Wales for the years 1880-1900 with corresponding numbers of Births per 1,000.

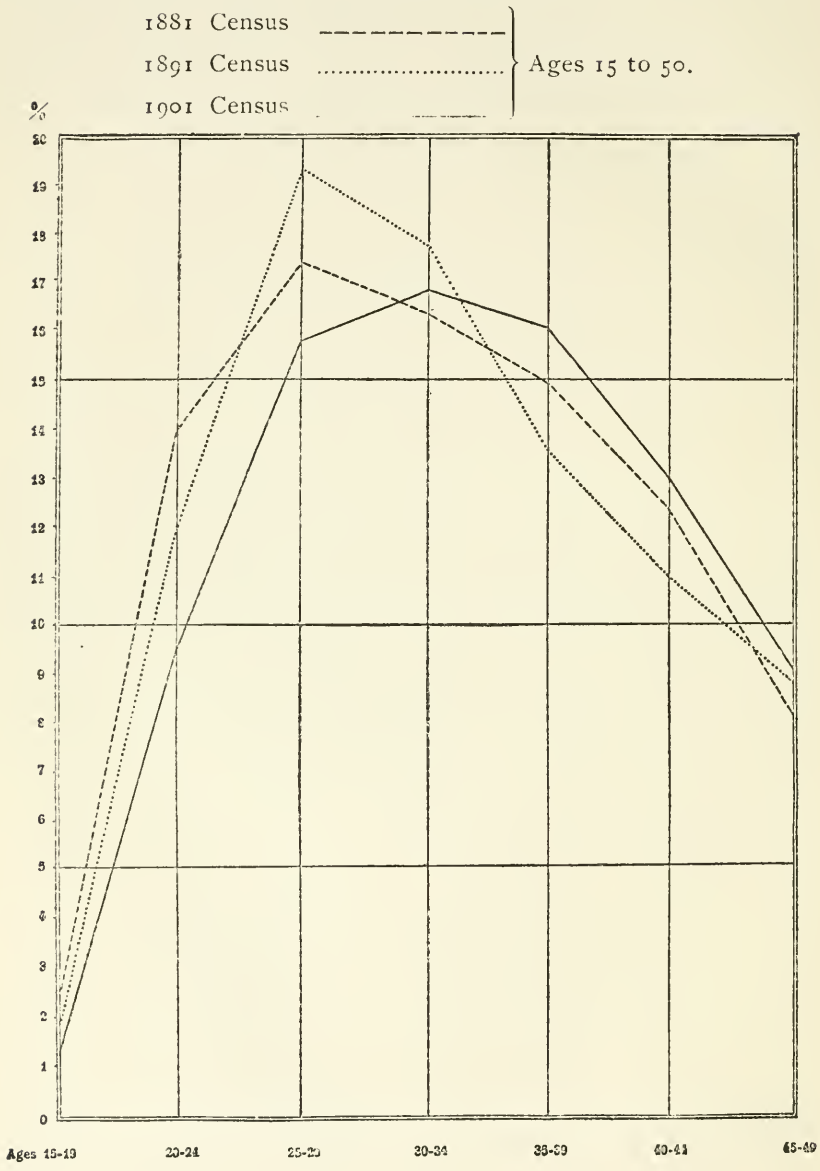
Year.	Mean Population at June 30th.	Number of Births.	Births per 1,000 of Mean Population.	Mean Female Population at June 30th.	Births per 1,000 of Mean Female Population.	Mean Female Population (ages 15-50) at June 30th.	Births per 1,000 of Mean Female Population (ages 15-50).	Mean Married Female Population June 30th.
1.	2.	3.	4.	5.	6.	7.	8.	9.
1880	728,958	28,162	38.63	329,919	85.36	159,639	176.41	104,834
1881	759,441	28,993	38.18	344,166	84.24	167,038	173.57	109,394
1882	791,197	29,702	37.54	359,025	82.73	174,777	169.94	114,150
1883	824,281	31,281	37.95	374,525	83.52	182,874	171.05	119,112
1884	858,748	33,946	39.53	390,695	86.89	191,347	171.41	124,290
1885	894,657	35,043	39.17	407,563	85.98	200,212	175.03	129,694
			38.52		**84.83**		**173.93**	
1886	932,067	36,284	38.93	425,159	85.34	209,488	173.20	135,332
1887	971,041	37,236	38.35	443,515	83.96	219,193	169.88	141,216
1888	1,011,645	38,525	38.08	462,663	83.27	229,349	167.98	147,355
1889	1,053,948	37,295	35.39	482,638	77.27	239,974	155.41	153,761
1890	1,098,023	38,960	35.48	503,475	77.38	251,092	155.16	160,446
			37.16		**81.25**		**163.87**	
1891	1,131,172	39,458	34.88	519,934	75.89	260,261	151.61	165,723
1892	1,152,693	40,041	34.74	531,706	75.31	267,326	149.78	169,492
1893	1,174,623	40,342	34.34	543,745	74.19	274,582	146.92	173,346
1894	1,196,970	38,951	32.54	556,055	70.05	282,036	138.11	177,288
1895	1,219,743	38,774	31.79	568,645	68.19	289,691	133.85	181,320
			33.64		**72.63**		**143.80**	
1896	1,242,948	36,506	29.37	581,520	62.78	297,555	122.69	185,443
1897	1,266,595	37,247	29.41	594,686	62.63	305,632	121.87	189,661
1898	1,290,692	36,222	28.06	608,150	59.56	313,928	115.38	193,974
1899	1,315,248	36,461	27.72	621,919	58.63	322,449	113.08	198,386
1900	1,340,269	37,146	27.72	636,001	58.41	331,203	112.15	202,898
			28.44		**60.34**		**116.87**	

TABLE III—*Concluded.*

Births per 1,000 of Mean Married Female Population.	% Mean Total Female Population (ages 15-50) on Mean Total Female Population.	% Mean Total Female Population on Mean Total Population	% Mean Married Female Population on Mean Total Female Population.	Number of Marriages.	Marriages per 1,000 of Females (ages 15-50).	Mean Married Female Population (ages 15-50).	% Mean Married Female Population (ages 15-50) on Mean Total Married Female Population	Births per 1,000 on Mean Married Female Population (ages 15-50).
10.	11.	12.	13.	14.	15	16.	17.	18.
268.6	48.4	45.3	31.8	5,572	34.90	88,667	84.6	317.6
265.0	48.5	45.3	31.8	6,284	37.62	92,492	84.5	313.5
260.2	48.7	45.4	31.8	6,948	39.75	96,482	84.5	307.8
262.6	48.8	45.4	31.8	7,405	40.49	100,644	84.5	310.8
273.1	49.0	45.5	31.8	7,482	39.10	104,986	84.5	323.3
270.2	49.1	45.6	31.8	7,618	38.05	109,515	84.4	320.0
268.8					**33.40**			**315.7**
268.1	49.3	45.6	31.8	7,811	37.29	114,240	84.4	317.6
263.7	49.4	45.7	31.8	7,590	34.63	119,168	84.4	312.5
261.4	49.6	45.7	31.8	7,844	34.20	124,309	84.4	309.9
242.6	49.7	45.8	31.9	7,530	31.38	129,671	84.3	287.6
242.8	49.9	45.9	31.9	7,876	31.37	135,267	84.3	288.0
255.1					**33.63**			**302.4**
238.1	50.1	46.0	31.9	8,457	32.49	139,467	84.2	282.9
236.2	50.3	46.1	31.9	8,022	30.01	142,178	83.9	281.6
232.7	50.5	46.3	31.9	7,749	28.22	144,942	83.6	278.3
219.7	50.7	46.5	31.9	7,666	27.18	147,760	83.3	263.6
213.8	50.9	46.6	31.9	8,030	27.72	150,632	83.1	257.4
227.3					**29.66**			**272.5**
196.9	51.2	46.8	31.9	8,495	28.55	153,559	82.8	237.7
196.4	51.4	46.9	31.9	8,813	28.84	156,544	82.5	237.9
186.7	51.6	47.1	31.9	8,888	28.31	159,588	82.3	227.0
183.8	51.8	47.3	31.9	9,275	28.76	162,690	82.0	224.1
183.1	52.1	47.5	31.9	9,996	30.18	165,850	81.7	224.0
189.2					**28.95**			**230.0**

NEW SOUTH WALES.

MARRIED FEMALES IN VARIOUS AGE GROUPS AS PERCENTAGES OF TOTAL MARRIED FEMALES.



When we come to the final test, viz., the proportion of births to the mean married female population between the ages fifteen and fifty (column 18), we find that the number per thousand has fallen from 317.6 in the first to 224.0 in the final year, a decrease of about twenty-nine and one-half per cent.; or if we group the figures in periods, the fall is from 315.7 per 1,000 to 230.0 per 1,000, a drop of 27.2 per cent. Now, inasmuch as the ratio of the married female population to the total female population has remained constant throughout the period (column 13, Table III), and the ratio of the married female population ages fifteen to fifty to the total married female population (column 17) has only fallen about three and one-half per cent., the natural deduction, apart from disturbing influences, is that the birth rate should have remained nearly constant when viewed as a function of the married females of ages fifteen to fifty. We find, however, from column 8 of Table III that this rate has fallen from 173.93 per 1,000 for the earlier period to 116.87 for the latest, a drop of nearly thirty-three per cent. Every deduction from the figures which I have supplied points to the one conclusion, viz.: that there has been a constant and serious decline in the birth rate. Can this decline be attributed to natural causes, or is it the offspring of that sinister suggestion which has of late years taken definite form in the public mind, and which appears to have led to the appointment of the Royal Commission now sitting? A contributory influence is suggested by the figures in the Table on page 7. The general teaching of the figures given above is that the decreased birth rate is largely due to the fact that the proportion both of married men and married women at the ages at which generation is most active has largely decreased. What are the causes which have conduced to this condition? The complete discussion of this question would lead me far beyond the limits of the present inquiry, and I can briefly indicate some of them:

1st. The increasing luxury of the age. Young women of the present day are not content to begin where their mothers began; on the contrary, they wish to begin where there mothers left off. The tendency to extravagance, the passion for ostentation, the ambition to make a figure in society, which are such distinctive features of present-day existence among females, make young men hesitate before assuming the responsibilities which married life imposes on them.

2d. The increasing employment of women in commercial avenues. In the old days, mothers and daughters both looked to marriage as the goal of existence, and the old maid shared with the mother-in-law the stigma of reproach. In later times, women have become wage earners, and the consciousness of the power to earn, with its attendant sense of freedom, has tended to make them impatient of the marriage yoke and the inconvenience of child-bearing. The grant of the sufferage will probably intensify this condition, and the world will no longer be ruled by the hand which rocks the cradle. Whether this consummation will make for the happiness and well-being of the community is beyond the scope of this inquiry.

3d. The practical cessation of immigration to the State. The discussion of this question would lead me into the hazardous arena of politics, so I merely indicate it, leaving its elaboration to others more qualified to discuss it.

Methods of Intervaluation Ascertainment of Reserves.

M. M. DAWSON.

Most banks balance their books daily and draw off a balance-sheet, placing a summary, which exhibits the transactions of the previous day and the condition of the bank at close of books, before the managers every business morning. In order to do this in a life insurance company, it is necessary, in addition to drawing a statement from the books, to ascertain the following which are not usually to be got from the books of account:

Reserves	Premiums Deferred
Premiums Outstanding	Accrued Interest.

It is much more common for life insurance companies to keep books with outstanding premiums and deferred premiums, than to compute reserve liabilities more frequently than annually. The latter is the chief difficulty in the way of a daily, weekly or monthly report as complete and definite as that of a bank, though to keep the accrued interest account up to date may also be no light matter.

The following modes of approximating or computing the reserves at intervaluation periods are known already:

1. By interpolation from the last two or more valuations.
2. By progressing individual reserves by monthly intervals.
3. By keeping a column of increases of reserves and a column for "mark-offs" in the valuation book. The reserve at any time is found by summing the former, the reserves on new issues and the reserves at last valuation and deducting the mark-offs. This process answers very well for a single computation; but the "increase" columns have to be summed again and again for frequent computations.
4. By keeping books like the above, but marking off the old reserve and entering all the new reserve, instead of the increase merely.

The modes which this paper will describe differ from these materially.

The aggregate mean reserves on December 31st are computed by American actuaries nowadays in the following fashion: The mean or midyear reserve is charged for each policy, according to the policy year entered upon; the sum of these is the aggregate reserve.

The effect of this is to treat each policy as if effected on June 30th of its year of issue; and on the whole it comes out well, for the policies really issued in January are balanced against those issued in December and so on.

Obviously the aggregate mean reserve on January 31st may be computed in precisely the same manner, viz.: By summing the mean reserves of all policies for the policy years respectively entered upon by them on January 31st. And so for any other date.

An analysis of what has taken place between December 31st and January 31st, shows changes of the following nature:

1. New policies issued in January, each charged with a first year's mean reserve.

2. Old policies, issued in January of previous years and renewed, changed from the mean reserves charged December 31st to the mean reserves for a period one year longer.

3. The reserves of policies, included in the December 31st valuation, cancelled, since that date by death, withdrawal, surrender or lapse.

4. The reserves charged and credited, on policies which are increased, reduced, changed, etc.

The only important amendment that is needed to the fourth method already in use, in order to render it a complete system, showing the condition from day to day, is to enter in a convenient book the sum totals of the changes which have been made that day in the valuation books. With this addition the mark-off column can be done away with entirely and the reserves be marked off by the time-honored and best method, i. e., drawing a line through the item. The entries of new reserves, as the policies reach their renewal date or are changed become the entries in the column for the next annual valuation, which are thus complete at close of books, December 31st, and should for the most part be footed as well.

The following would pass through the valuation clerk's hands daily:

A list of the new issues of the day giving numbers, amounts, ages and plans (and as a check, amounts of premiums) with any unusual particulars.

A similar list of policies reaching their anniversaries on the day, renewal receipts being outstanding or renewal premiums already paid.

A similar list of policies discontinued or changed in any way.

From these lists the clerk corrects the valuation book, entering upon another book the amounts of reserve charged on and off, as follows:

On.			Off.		Totals.	
New Issues.	Renewed.	Changed.	Renewed.	Discontinued.	On.	Off.

At close of business, these columns may be footed and the footings be carried into a Reserve Ledger; or the footings may be carried forward from day to day.

In either case, the reserve account beginning on January 1st with the reserves according to the annual valuation, should close on December 31st with the same total as brought out by the valuation.

More elaborate processes may occasionally be needed in practice, especially when it is desired to keep the accounts of deferred and outstanding premiums, with the reserves.

(*Presented by Mr. Robert Henderson.*)

Note on Lubbock's Formula.

JOHN A. McKELLAR.

The problem of which Lubbock's Formula supplies a solution is the approximate evaluation of the finite definite integral $\Sigma_0^{mn-1} u_{\frac{x}{n}}$ when the equidistant values $u_0, u_1, \ldots u_{m-1}$, are known. This can most easily be done by the method of the separation of symbols, a rigorous proof of which is contained in standard works on Finite Differences.

The principal difficulty to be found in performing the operation is the determination of the coefficients in the expansion of the function $\frac{\Delta}{(1+\Delta)^{\frac{1}{n}}-1}$. The method usually employed depends upon the development of this function by means of Maclaurin's Theorem, but a much simpler and more easily remembered process is that known as the method of Indeterminate Coefficients, for a general treatment of which the Algebra of Professor Chrystal, Vol II, Chapter xxvi, may be consulted.

Adopting the notation of the Calculus of Finite Differences, and separating symbols, we have:

$$\Sigma_0^{mn-1} u_{\frac{x}{n}} = \{1+(1+\Delta)^{\frac{1}{n}}+(1+\Delta)^{\frac{2}{n}}+\ldots\ldots\ldots+(1+\Delta)^{\frac{mn-1}{n}}\}\, u_0$$

$$= \frac{(1+\Delta)^m-1}{(1+\Delta)^{\frac{1}{n}}-1} \cdot u_0$$

$$= \frac{\Delta}{(1+\Delta)^{\frac{1}{n}}-1} \cdot \frac{(1+\Delta)^m-1}{\Delta} \cdot u_0$$

$$= \frac{\Delta}{(1+\Delta)^{\frac{1}{n}}-1} \cdot \frac{1}{\Delta}(u_m-u_0)$$

$$= \frac{\Delta}{(1+\Delta)^{\frac{1}{n}}-1}(U_m-U_0) \qquad \text{(where } \Delta\, U_x = u_x\text{)}$$

Let $\frac{\Delta}{(1+\Delta)^{\frac{1}{n}}-1} \equiv A + B\Delta + C\Delta^2 + D\Delta^3 + E\Delta^4 + \text{etc.}\ldots$

then, by cross multiplication

$$n \equiv \left(1 - \frac{n-1}{2n}\Delta + \frac{(n-1)(2n-1)}{6n^2}\Delta^2 - \frac{(n-1)(2n-1)(3n-1)}{24n^3}\Delta^3 + \ldots\right)\left(A + B\Delta + C\Delta^2 + D\Delta^3 + E\Delta^4 + \ldots\right)$$

Distributing and equating the coefficients of like powers of Δ the values of A, B, C, D,....&c., may be determined as follows:

$$A = n$$

$$B = A\,\frac{n-1}{2n}$$

$$= \frac{n-1}{2}$$

$$C = -\,A\,\frac{(n-1)(2n-1)}{6n^2} + B\,\frac{n-1}{2n}$$

$$= -\,\frac{(n-1)(2n-1)}{6n} + \frac{n-1}{2}\cdot\frac{n-1}{2n}$$

$$= -\,\frac{n^2-1}{12n}$$

$$D = A\,\frac{(n-1)(2n-1)(3n-1)}{24n^3} - B\,\frac{(n-1)(2n-1)}{6n^2} + C\,\frac{n-1}{2n}$$

$$= \frac{(n-1)(2n-1)(3n-1)}{24n^2} - \frac{(n-1)^2(2n-1)}{12n^2} - \frac{(n^2-1)(n-1)}{24n^2}$$

$$= \frac{n^2-1}{24n}$$

Similarly the coefficients of the higher powers of Δ can easily be found.

Substituting the above values of the coefficients we have

$$\frac{\Delta}{(1+\Delta)^{\frac{1}{n}}-1} = n + \frac{n-1}{2}\Delta - \frac{n^2-1}{12n}\Delta^2 + \frac{n^2-1}{24n}\Delta^3 + \text{etc.}.....$$

and therefore

$$\Sigma_o^{mn-1} u_{\frac{x}{n}} = \{n + \frac{n-1}{2}\Delta - \frac{n^2-1}{12n}\Delta^2 + \frac{n^2-1}{24n}\Delta^3 + \}\,(U_m - U_o)$$

$$= n\,(U_m - U_o) + \frac{n-1}{2}(u_m - u_o) - \frac{n^2-1}{12n}(\Delta u_m - \Delta u_o)$$

$$+ \frac{n^2-1}{24n}(\Delta^2 u_m - \Delta^2 u_o) + \text{etc.}$$

$$= n\,(u_o + u_1 + u_2 + \ldots\ldots\ldots\ldots + u_{m-1})$$

$$+ \frac{n-1}{2}(u_m - u_o) - \frac{n^2-1}{12n}(\Delta u_m - \Delta u_o) +$$

$$\frac{n^2-1}{24n}(\Delta^2 u_m - \Delta^2 u_o) + \text{etc.}$$

Address of the President,

ISRAEL C. PIERSON,

in opening the Annual Meeting May 19th, 1904.

To-day we celebrate the fifteenth Anniversary of the organization of the Actuarial Society of America. At the annual meeting on May 18th, 1899, as Secretary, I gave a review of its history and work for the first ten years, and it seems not inappropriate for me at this time to present a brief account of the record of the Society during the past five years, which in some respects have been eventful.

MEMBERSHIP.

On May 18th, 1899, there were 105 members and 7 associates. The total membership at this date is:

Members	122
Associates	33
Total	155

Of the thirty-eight charter members on April 25th, 1889, twenty-nine are now living, six having died during the first ten years and three during the last five years: Messrs. Gould, McCabe and Starr. Of the seven ex-Presidents we are very happy to say only one is absent, Mr. Homans.

MEETINGS.

The Society has held nine meetings, six in New York and one each in Hartford, Boston and Washington, D. C., the Fourth International Congress of Actuaries having taken the place of the fall meeting of 1903. The decennial meeting in 1899 was signalized by a reception given to the members of the Society on Thursday evening, May 18th, by the officers of the Metropolitan Life Insurance Company and by a banquet at Delmonico's on Friday evening, May 19th, when the presidents

of the Life Companies and others were present as guests of the Society, Mr. Bloomfield J. Miller, President of the Society, being in the chair.

INTERNATIONAL CONGRESSES OF ACTUARIES.

At the Third International Congress of Actuaries in Paris, June 25th to 30th, 1900, the Society was represented by seven members in attendance and by forty-seven subscribing adherents from the United States and Canada. In pursuance of the action of the Council and of the Society an invitation in English, French and German was presented to that Congress, and to the Permanent Committee on International Congresses of Actuaries, to hold the next Congress in New York in September, 1903. The invitation was cordially accepted and the Council of the Society was appointed "The Organizing Committee of the Congress." Accordingly the Fourth International Congress of Actuaries convened in New York under the auspices of this Society on August 31st to September 5th, 1903. An account of that Congress appeared in No. 30 of our *Transactions* but the full proceedings, an immense amount of material, is now being edited by the General Secretary of the Congress, Mr. Tatlock, and will be published in due time.

PRIZES.

In 1900, Mr. D. P. Fackler to encourage independent investigations, and the pursuit of studies for the general benefit of the profession, offered a prize of $150 to be known as the "Associates Triennial Prize" for the best paper on any proper actuarial topic. This offer by Mr. Fackler was accepted with many thanks and with special recognition of his continued interest in this direction. A Committee of four members of the Council formulated rules for the competion and the President appointed as a Committee to award the prize, Messrs. St. John, Macaulay, C. C. Hall, Nichols and Torrey; they will make a report at this meeting of the Society.

PUBLICATIONS—*Transactions.*

During the five years there have been published ten numbers of the *Transactions,* containing 997 pages. The No. 30, published by vote of the Council, includes, as before stated, a brief account of the proceedings of the Fourth International Congress of Actuaries, with the opening addresses at the Congress,

and an address by Mr. Tatlock before the National Association of Life Underwriters, Baltimore, Md., October 15th, 1903: "The Recent Actuarial Congress. A Review of the Topics, Discussed Thereat, Which are of Interest to those Engaged in the Field Work of Life Insurance." During these five years forty-two members have presented one or more articles each, on actuarial subjects either practical or technical, and many others have taken part in the discussion of them.

PUBLICATIONS—*Specialized Mortality Investigation.*

The paper, "On the Objects to be Attained in Future Investigations of Mortality and Death Loss," presented by Mr. McClintock at the Third International Congress of Actuaries in Paris was read at the meeting of the Society, October 25th, 1900, and aroused so much interest that on motion a Committee consisting of Messrs. McClintock, Miller, Van Cise, Weeks and Wells was appointed to consider the question of collecting the mortality experience of American Life Companies in reference to the special points of inquiry. Mr. J. K. Gore was subsequently added to the Committee. The President, Mr. Macaulay, in putting the motion as to the programme for the investigation, said, "It is opening a new line of usefulness, and one which will place all the companies of the country under obligations to our Society, and it has this other attraction, that it is something new, no other Actuarial Society in the world having ever adopted a line of investigation like this." At the meeting of May 21st, 1903, Mr. McClintock made a preliminary report, which by vote of the Society was printed for immediate distribution to the members. The Committee prosecuted the work with such expedition and success that it was completed and printed by August 1st, 1903, a small quarto volume of 479 pages. A copy of it was presented as a souvenir to each member of the Fourth International Congress of Actuaries of 1903.

PERMANENT QUARTERS.

Many members have been anxious for some time to have permanent quarters for the Society. Within the past few months the Council has made an arrangement with the Metropolitan Life Insurance Company whereby we have secured rooms for the Library and other purposes, and the use of a large hall for the meetings of the Society.

DEATHS.

Within the past five years we have lost the following members by death : A. J. Finlaison, September 17th, 1900; Augustus F. Harvey, August 28th, 1900; W. L. Daughtrey, Jr., an Associate, April 30th, 1901; William O. Gould, December 31st, 1901; William E. Starr, January 13th, 1903, in his ninety-first year; James Meikle, John B. Tennant and Arthur F. Burridge. While the loss of these members is a bereavement, we were all so much attached to Mr. Starr and hopeful that he would live to celebrate his centennial, that his death was a special sorrow. It is singular that the last three in the list given above were members of the Fourth International Congress of Actuaries as delegates from the Institute of Actuaries. A minute concerning each one has been prepared and will be read at this meeting and reprinted in the *Transactions*.

WORK OF OFFICERS, COUNCIL AND COMMITTEES.

We keenly appreciate the services and active interest in the work and welfare of the Society on the part of the officers and members, and we owe hearty thanks to them ; to the Presidents, Mr. Macaulay and Mr. Ireland, in their official positions ; to Mr. Tatlock for his labors as Secretary of the Society, and for his efficient although stupendous work as General Secretary of the Congress; to Mr. Welch as a faithful treasurer; to the members of the Council, who have been constant in their attendance at meetings and in devotion to their duties ; to Mr. Fackler for his offer of prizes ; to Mr. McClintock for proposing the Specialized Mortality Experience, and for devising the general plans for the International Congress of Actuaries; to the Organizing Committee of the Congress ; to the Committee on the Specialized Mortality Experience and to the Committees who have prepared the papers and conducted the examinations for admission as Associates or Members of the Society.

As has been said, we are commemorating the fifteenth Anniversary of the Society. Often during these years the dinner, the social intercourse and the postprandial addresses have been among the most enjoyable and beneficial experiences of the meetings.

An important part of this celebration will be the dinner this evening when the six ex-presidents will be the after dinner speakers. Mr. Fackler, "The Founding of the Society;" Mr.

St. John, "The First President of the Society, Mr. Homans;" Mr. Macaulay, "The Work of the Society During the Past Fifteen Years;" Mr. Miller, "Promotion to Office of Actuary and Higher Honors;" Mr. McClintock, "International Congresses of Actuaries;" Mr. Ireland, "The Benefits of Association." I believe that this dinner will be one of the most interesting in our history and hope that every member attending these business sessions and as many others as possible will be present.

Although you were good enough a year ago to elect me to the office of President of the Actuarial Society of America, this is the first time that I have had the opportunity and the honor to preside at a meeting. Thanking you heartily for your favor and kindness during my term of office now closing, I trust that this quin-decennial meeting will be profitable in the reading and discussion of papers and that it and our social re-union will be an inspiration and a happy memory for the long future.

Abstract of the Discussion of Papers read at the Previous Meeting.

On Extra Mortality Considered in Connection with Deferred Dividend Contracts—Hugh W. Robertson.

Page 383, Vol. VII.

Written Discussion.

Mr. Hutcheson: In the paper which Mr. Robertson submitted a year ago he showed the effect which various assumptions as to the rate of mortality during a dividend period would have on the fund in hand at the end of that period, for groups of policies all paying the premium for standard risks. The assumption he made regarding mortality was that the same aggregate number of deaths would take place in the various groups during a twenty-year dividend period, but that the deaths would be distributed in different manners—in one case the extra mortality coming immediately after issue, in another group in the middle of the period, in another at the end, and so on.

As to his results; if we compare the curves for groups B and F, for example, in the diagram opposite page 385, we can see at once that the fund for group B at the end of the period will be much smaller than that for group F for these three reasons; (1) because the claims will be paid earlier, (2) because fewer premiums will be received, and (3) because less interest will be earned in group B than in group F. In the same manner we can, in a general way, arrive at the conclusion that the results of the various other groups will bear the relations shown in Table XXII (Page 417). The actual results in dollars and cents are of little importance because they show the combined effect of various assumptions as to mortality, lapse, interest, etc., which are never likely to occur.

I presume the object Mr. Robertson had in view was to ascertain whether the surplus at the end of the dividend period could be depended upon to pay for the extra mortality experienced, or whether it would be necessary to make deductions from the amounts payable as death claims.

Mr. Robertson did not tell us what rates of mortality the various life groups would follow after the twenty-year period, and that is an all important point and one which we must know in order to be able to decide what part of the fund must be held as reserve—any balance, of course, being surplus. If we look at the curve for group F, for example, it is evident that the mortality after the end of the period will be so far above the normal that a large extra reserve will be required, and this heavy mortality will probably be accentuated by reason of the fact that many of the healthier lives in the group will surrender their policies for cash at the end of the dividend period—thus reducing the average vitality of those continuing.

Mr. Robertson gives us no hint as to how he would assess the extra, but it is clear that, owing to the heavy mortality (*during* the period in *some* cases, and *after* the period in *others*), the extra mortality cannot, in the case of life policies, be paid for out of surplus over and above necessary reserves. The fund in several of the groups, in fact, at the end of the period is less than the American three per cent. reserve.

If each life group is to pay for its extra mortality, part at least of the extra must therefore be paid by making deductions from the death claims. What these deductions should be will depend upon the incidence of the extra mortality, and upon the cash surrender values guaranteed. Given the mortality table which any class of impaired risks will follow, the amount of lien can be ascertained by equating the present value of the liens to the present value of the difference between the net premiums by the sub-standard and standard tables—all calculated by the sub-standard table. The incidence, amount and duration of the lien should, of course, be adjusted so that the *actual* reserves from time to time will be sufficient to enable the company to give whatever cash and other policy guarantees it may decide upon.

Mr. Weeks: Mr. Robertson has done a piece of useful work, in thorough and painstaking fashion. The method he has used is a common sense method, and the relative approximative results which it brings out are valid.

Two points of practical value occur to me as inferences from his paper: first, the immense superiority of the endowment plan over the life plan for handling sub-standard risks; and, second, the ease with which, on the endowment plan, an effective equivalence can be established between impairments of very different natures. It would be a simple process, for example, taking the four types of impairment which Mr. Robertson calls A, B, C and D, to find for the several types what initial degrees of impairment would produce the same financial result in all four of the cases. Having done this, cases of all the different types can, in practice, be put together into common dividend classes—assuming, of course, a fair degree of confidence in the diagnosing of the degree and incidence of the impairment.

Mr. Robertson gives us seven distinct types of impairment, and I have been much interested in examining them. The first two are the standard "garden" varieties—so to speak—the kind which has an arithmetical constant, and the kind which has a geometrical constant. His third type is a very probable "hybrid" of the first two. In his fourth type he gives us what I might call—if I may go on in the horticultural vein—a "double-flowering" variety—a rarity, but still a natural production. He then presents us with three monstrosities, two of which he admits are artificial, but one of which, his type E, he thinks is genuine.

I had the curiosity to look into the recent Specialized Investigation, to see whether there is any instance there which bears out Mr.

Robertson's view of the existence of groups in which the incidence of the excess mortality falls with extreme heaviness in the latter part of a twenty-year period. Looking through the list, to find the group which shows the greatest excess of the ratio for the sixth to thirtieth years over the ratio for the first five years, I found that this was Class 61—the heavyweights with pendulous abdomen. I took out the exposures and deaths of this class for eleven ages of entry, thirty five to forty-five inclusive, and for the successive insurance years to the twentieth, inclusive; I then calculated the expected deaths for each insurance year according to Mr. Robertson's expected mortality among endowments, and compared the expected deaths with the actual for the successive insurance years. The number of deaths is not large enough to give a regular distribution, but the results can be fairly interpreted as a case of Mr. Robertson's type D, with an excess mortality beginning at eleven per cent., and increasing eleven per cent. every year to the twentieth, inclusive. I should not expect to find, in practice, any group showing the traits of his type E, in which he gives us 100 per cent. mortality the first year, 110 per cent. the tenth year, and *378 per cent.* the twentieth year.

A NEW VALUATION FORMULA—M. M. DAWSON.

PAGE 418, VOL. VII.

WRITTEN DISCUSSION.

MR. WELLS: The formula, or rather the method of valuation, proposed by Mr. Dawson, is ingenious, and I think may sometimes be of considerable value. I do not think, however, that such a valuation can be properly called "a strictly net premium valuation;" certainly not a net premium valuation on the basis of a given table. It does not follow, however, that it might not be for some purposes even more valuable than a net premium valuation. If the "ultimate" table "represent conservatively what future experience will be" after a given time, and the "select" table represent conservatively what it will be meanwhile, and the contract or office premium be such that the "net premium" as derived from the "ultimate" table can be relied on as wholly applicable to the payment of present claims and the accumulation of a reserve for future claims, the reserve found (discarding negative values, if any) will be sufficient for its purpose. But the reserve as found by Mr. Dawson's method differs from the net premium reserve based on the "ultimate" table only in that the former anticipates at the inception of the contract all mortality gains represented by the variation between the two tables, while under the latter method they appear as profits when earned. The anticipation is not, however, a valid objection, if, as assumed, the select table "represent conservatively what future experience will be." After the inception of the contract, so long as the two tables differ, the reserve

as computed by Mr. Dawson's method increases more rapidly than the net premium reserve by the "ultimate" table. This because the receipt of the net premium by the "ultimate" table is assumed, while only the lower mortality of the "select" table is to be provided for. The immediate gain, at the expense of the future, is, as pointed out by Mr. Moir in his paper before the Fourth International Congress of Actuaries, the negative value of the contracts at their inception.

In developing his formula Mr. Dawson so obviously had in mind the finding of a method of valuation which would permit of a larger expenditure for obtaining new business by reducing the reserve on such business, that no apology is required for a word as to the availability of the method for that purpose. I do not think Mr. Dawson's method will serve that purpose for two reasons: first, the laws of the States in general require, either specifically or as a minimum requirement, a net premium valuation upon the basis of a given mortality table, which the proposed valuation is not: second, the reduction in the reserve is not large enough to answer the demands of the companies seeking such relief, unless, indeed, they use this method of valuation to still further increase the sum temporarily released for expenses by the so-called First Year Term plan.

Oral Discussion.

Mr. Sheppard: I suppose Mr. Dawson does not mean that we are to have one set of reserves to be used in calculating the liabilities of the company and another set to be used for the calculation of gains from different sources and the dividends that would result therefrom.

If we take aggregate tables all through, the profit from selection comes in at the commencement, and could be held over, if necessary, and distributed later; if the select tables be taken, then that profit disappears.

It seems to me that theoretically a perfect system would be to have select tables all through in the calculation of profits and dividends. Selection is an essential feature in life insurance; and I do not think it ought to be mixed up with the question of the heavy initial expense which we pay at present, but from which we may at some future time be relieved, that is, first years' commissions should be excluded from the discussion altogether.

In regard to questions of aggregate, select, the American and ultimate tables, which are all involved in this problem, Mr. King made an investigation some years ago and traced the reserve by calculations based on the Analyzed tables and then on the H^M and H^M(5) combined. The result of that was to give a greater reserve by the Analyzed tables than by the H^M and H^M(5) until between the 25th and 30th year, and after that they ran pretty well parallel except that the Analyzed gave slightly smaller values. The reason why the H^M and H^M(5) have been used in England is that the companies desired to

strengthen their reserves and it is necessary to value reversions and premiums separately, and it is possible to do so only with very great labor in the case of Analyzed tables. When it was found that the HM and HM(5) ablest combined gave as strong reserves as the Analyzed tables, it was usually adopted ; but I do not think it is justifiable to quote, as Mr. Dawson does, the HM and HM(5) tables for the purpose of diminishing the reserve, when it was used in England for the purpose of increasing it. Then Mr. Dawson says the American table is an ultimate table. I think that is very doubtful. In the case of a company doing a very large business, we find that the percentage of death losses from insurance may be as little as sixty per cent. Now, Mr. Dawson's reserves are a mixture of select and ultimate. Assuming for the moment that the American table is an ultimate table, the net premium that he uses must be compiled from the American table, and therefore, so far as the premium is concerned, he must neglect altogether the experience of the years when selection is strongest, in order to get the reserves under these early years. If we had to use mixed figures at all, we ought to have a mixture of select and aggregate, using the select values of the reversions and annuities.

Dr. Sprague in one of his papers has said, if selection be taken into account in order to get the reserve, we ought undoubtedly to have used net premiums derived from the aggregate table ; but when we consider that, after all, competition regulates the premium within certain limits, I think we are justified in disagreeing with that and saying that the select net premium is one that should be used,—in which case we should get the reserve entirely calculated on a select basis, which is larger than the present one,—which is not what Mr. Dawson wishes.

Finally, I should like to say that, if the Actuarial Society is ever called upon to recommend any change in legislation, I think it ought to be in the way of stronger and not of weaker reserves.

MR. MACAULAY: As has already been pointed out, there are three varieties of tables that may be used by the companies in the calculation of net premiums and reserves—the aggregate, the select and the ultimate tables.

If I understand Mr. Dawson aright, when he applies the word ultimate to the American and Combined tables, he does not refer to the manner in which they were prepared, because if so, they are aggregate tables. His point is that if any mortality table provides for death losses which will be in excess of what may reasonably be expected among assured lives after, say, the expiration of five or ten years, such a table is an ultimate table, no matter how it was prepared. Of these three kinds of mortality tables, I believe that the one most suitable for the calculation of the reserves is the ultimate one. If the table used as a standard is based on assurance experience after eliminating say the first five years of the life of the policies, the premiums and reserves thus found would be scientifically correct, but would,

nevertheless, allow the companies to place the saving in mortality resulting from selection against the initial expense of the policies, and I think that is thoroughly fair.

But the point I wish to particularly emphasize is this, aggregate tables like HM or the Actuaries' or the American table to the extent that it is an aggregate table, give heavier reserves for the business of progressive companies than those that would be calculated by the use of the ultimate table like the HM(5). The reason is simple. The proportion of newly selected lives is not spread over all ages. As a rough approximation, we may suppose that in an aggregate table at age twenty, ninety per cent. of the exposures are on lives whose policies have been less than five years in force ; at thirty perhaps seventy per cent. ; at forty perhaps fifty per cent. ; at fifty perhaps twenty-five per cent. and at sixty possibly ten per cent. The percentage of newly selected lives thus brought into the exposures in connection with any aggregate table is a varying quantity, being heavy at the younger ages and light at the older ages. The result is to abnormally depress the mortality at the younger ages, that depression gradually diminishing with advancing age, and disappearing entirely at ages of seventy or over. In other words the normal mortality is diminished by the cutting off of a wedge, so to speak, the broadest part of which is at the younger ages of life, and the point at about age seventy. If we take an ultimate table like HM(5) as the true basis for the calculation of net premiums and reserves, it follows at once that the reserves brought out by any aggregate table are unnecessarily heavy, for, as we all know, the amount of the reserve does not depend so much in the case of most policies upon the rate of mortality, as it does upon the sharpness of the mortality curve. Anything which tends to accentuate and steepen the curve, such as depressing it at the early ages and increasing it at the higher ages, increases the reserve. If the reserves brought out by an ultimate table like the HM(5) be sufficient, as I think we must admit, it follows that a company is doubly safe in taking the HM Table. Such a company is, in fact, already setting aside more than is really necessary.

The new British Experience increases still more, and to an altogether undesirable degree, the sharpness of the mortality curve as compared with what is normal or ultimate. You will remember that the basis upon which that experience is based is the policies issued by the contributing British companies after 1863, and also the experience after that date on policies issued prior to that date. In other words, the policies issued, we will say, in 1853, are not included in that experience for the first ten years of their lives at all, but are included from their tenth year onwards. The result is to add a great number of lives not recently selected without a corresponding increase in the number of newly selected lives, the early years of the policies not being included in the experience. As the additions of the lives assured under these old policies occur, of course, chiefly at the older ages, the per-

centages of newly selected lives at those higher ages are still further reduced as compared even with an aggregate table like the HM or the Combined, to say nothing of an ultimate table.

I have not the exact percentages with me, but I may illustrate the principle in more detail. If we take the HM Table, at age twenty, ninety per cent. of the lives are possibly on policies of less than five years' duration. In the British Experience the percentage was probably about the same, the addition of the lives assured before 1863 would hardly affect the mortality at age twenty at all. When we come to age thirty, however, there are enough additional lives added of more than five years in force, to increase the percentage of old lives (or decrease the percentage of new lives), and consequently to increase the mortality at that age.

At age forty, the percentage of newly selected lives is still further reduced from perhaps fifty to thirty or thirty-five per cent.; at age fifty, from say twenty-five to perhaps ten; and at age sixty still more. The effect of thus reducing the number of newly selected lives at the higher ages is to increase the mortality at the higher ages more proportionately than at the younger ages; in other words, to still further steepen the curve.

If the fact that aggregate tables already give a naturally steep curve is objectionable, to still further arbitrarily steepen that curve is a very serious matter; the result is to give abnormal reserves, having, I fear, but little scientific basis. This peculiarity of the new British table constitutes a defect which will seriously militate against its general acceptance and future usefulness.

MR. FACKLER: Mr. Sheppard referred to the American table as an aggregate table. He is probably not as familiar with the literature connected with the American table as the most of us, and I would say that it is an ultimate table and that the intention was to eliminate the influence of the first five years. I think that a little study of Mr. Homans' statement in connection with the American table will show this.

MR. MILLER: I think from my observation and study that, for American lives, the American Experience table of Mortality is a remarkable success viewed as, what is technically called, an ultimate table. It is not in any way a simple, blind table made up from certain particular observations which are faulty, and need adjustment. The table after it was made was carefully and, as it seems to me, very judiciously adjusted, and comes about as near representing what may be called the normal rate of mortality in this country, among lives belonging to that section of society which are likely to take insurance policies, as we would be likely to get.

Now, I have noticed by very careful comparisons and observations that when we take the companies which have published their mortality statistics made from unadjusted observations and compare

the results of those observations with the American table, that the American table seems to be almost perfect as a good, safe guide for the computation of reserves and premiums on American lives. If we take the death rate in the first year of insurance as experienced by our companies which have published their experience, and they comprise the oldest and the largest companies, there is found a wonderful parallel between the actual death rate for all ages at entrance and the death rate by the American table. I have found that taking the first year of membership the actual death rate from ages twenty-five to seventy has been almost parallel with the corresponding lines representing the death rate by the American Experience table. I have found that as selection died out the same law held good very closely indeed, and, when selection had practically died out altogether after the fifth year, I have still found that the line representing the actual death rate in American life insurance companies has run along very parallel to the American Experience death rate line and has been about five per cent. below it. It has continued at about five per cent. below the American table for almost all durations of policies. Therefore, I think that the American table is a very safe table, and that it represents as nearly as may be the ultimate death rate among American insured lives with a margin of about five per cent. for safety.

There are certain periods in the history of policies issued by American life insurance companies, when the tendency is for the actual death rate to run up to the full American death rate, and perhaps a little over. These periods range between the tenth and the twenty-fifth policy years. After that there seems to be an improvement in the experience of the companies. The line representing the actual death rate runs down quite sharply below the American experience. Why it does so is a matter for consideration and debate. It may be because the American death rate is too high for extremely old ages. On the other hand it may be that when policy-holders get to be seventy or seventy-five or eighty years of age, the tendency of the unsuccessful and the poor is very, very strong to surrender their policies and get what value they have for their own support. Their families no longer need protection as much as they do. It may be that surrender values at the higher ages of life, instead of exercising an unfavorable selection to a greater or less extent against the company, really exercise a very beneficial one. At all events, in my experience, I have often been struck by policy-holders ranging between sixty and seventy years of age who wanted to surrender their policies. I have sometimes advised them that they were foolish to sell; that their condition of health was such that they ought to hold their policies. But the answer has been substantially in all cases, "The value of those policies is worth more to me than it is to anyone dependent on me, and I want the money; I need it."

On the whole, I am very strongly convinced that the more the actual mortality is compared with the theoretical death rate of

the American table, the more satisfactory will the American table be to the careful student of the conditions which prevail with American life insurance companies.

MR. HANN: I am inclined to think that it will be found as time goes on that the American table is, if anything, weak at the old ages, and therefore when we compare our results with that table, it makes the mortality among old lives appear perhaps larger than it really is. I believe in taking an aggregate table when carefully prepared and framing our premiums from it, and when once framed I believe in adhering to those premiums. I do not think it is wise for us to pare off from the first, or from any other year, a portion of the reserves in order to meet the expenses for the corresponding periods. You will find that when that is done, the demand upon the expenses will increase. I believe that we should aim for a premium basis fixed on facts as near as possible, and not spend the whole that we appear to gain from mortality in the first five years.

MR. MOIR: I wish to emphasize very strongly one remark made by Mr. Macaulay that, while it is *desirable* to carry very strong reserves and to raise the reserves gradually to the highest possible pitch, it does not follow that it is in any way *necessary* that they be so increased.

A policy taken out under ordinary conditions, and carrying the usual reserves, is not a paying proposition until it has been on the books of the company from two to three years at least, and occasionally not until the fifth year. This position has been recognized by most of the legislatures (noticeably in the State of Massachusetts) by excluding the first two years of the policy from surrender value calculations.

The American table of Mortality is now the recognized standard in the United States, and it is generally admitted that this table approaches very closely to the actual mortality experience of companies after the early effects of medical selection have passed. Whether it was originally an aggregate table or an ultimate table is of little consequence. The mortality represented is understood to be about the same as the ultimate mortality now experienced by American companies. This table is used as the basis for premium calculations, and the same net premiums should properly be introduced in the valuation of the policies. On the other hand, select annuities in the early years represent the actual facts according to the experience of companies, and therefore they may be employed for valuing the future net premiums. This is the general principle.

The benefits from the selection of healthy lives arise directly as the result of good management of the company. If any company were not carefully managed, and if the lives were not judiciously selected, no profit would result from this source. Part of the expense of placing new business is undoubtedly chargeable against the advantages thus derived, and much can be said in favor of charging the entire amount against the saving in mortality from effective selection.

MR. WEEKS: It seems to me that Mr. Dawson's formula is a legitimate formula. In the matter of reserves the theory ought to follow the facts and not precede them, and the theory of level net premium reserves ignores the fact that expenses are higher in the first year than in subsequent years. Therefore, it would be legitimate to construct reserves taking that into account, and Mr. Dawson has accomplished this by an ingenious and correct method.

MR. VAN CISE: I suppose we could theorize here for a long time about this matter and maybe, as Mr. Weeks says, it is a matter of theory, but as a matter of practice, our reserve liabilities are calculated in accordance with law. Nearly all of our forty-five states have standards of liabilities which are not likely to change, and they make no distinctions as to the age of policies, whether more or less than five years, and the whole effect of this, as a matter of practice, would be to weaken the company, and in view of the liberal contracts and the cost of the business, it is certainly inadvisable to weaken the standard which we have now.

Many of the American companies doing business throughout the whole of the United States are working in many localities where the mortality is higher than that of the American table. A large portion of the Southern States show a larger death rate than is shown by the American table.

In view of that fact, we should not adopt a standard which is lower, and if we should only reserve these smaller amounts for a large volume of business of less than five years of age, it would certainly weaken the companies. It might not hold, not only against the possibility but the certainty, that a larger part of the United States would have a mortality higher almost than the American table.

MR. MACAULAY: In years gone by we have heard a great deal about the necessity or desirability of making some reduction from the ordinary reserves on account of the heavy additional expense in connection with the first year of every policy. We are not now discussing that aspect of the matter.

On the other hand, however, we must not forget that while the companies have to face a heavy initial expense in connection with the policies, they also have the right to expect a favorable mortality, which will be a considerable saving to them. I do not think that we are any more justified in ignoring that saving in mortality than in ignoring the expense that the companies are put to in the earlier years. I think that any person who claims that the companies should ignore the expense in connection with the first policy year must also, in fairness, take the position that the companies should ignore the other side of the question, the saving in mortality during the early years. The rule ought to work both ways or not at all. If the companies were to do business in such a way as to simply eliminate the benefits of medical selection, and thus get a mortality equal from the commencement of the policies to the ultimate table, I think we

would all agree that we would be acting safely and correctly in providing just the reserves called for by that ultimate table. The fact that the companies save a certain amount in mortality during the early years has just as much right to be taken into account as the fact that they have to face a heavy initial expense. If the reserves are not to be reduced because of the heavy initial expense, neither should they be increased because of the saving in mortality.

But even if we ignore the question of expense altogether, the ideal standard for the calculation of premiums is, I think, an ultimate table. We may, if we choose, adopt a table which calls for heavier reserves than that, as we do when we adopt an aggregate table, but, we must admit that when we thus adopt an aggregate table, we adopt a standard that brings out unnecessarily heavy reserves for most policies. I do not think we have any right to exact higher reserves than those called for by an ultimate table.

I would like also to emphasize Mr. Moir's remarks that we have to draw a sharp distinction between reserves that are necessary and reserves that are not necessary but still expedient. It is necessary to have legal standards which will be as high as the requirements of safety, but no higher. To establish an unnecessarily high legal standard of solvency is as unwise as if the Government, upon a coast studded with rocks dangerous to navigation, were to build artificial rocks away out in the ocean in order to keep vessels away from the real rocks. That is what an unnecessarily severe standard of solvency is like. It may wreck companies but cannot do good. On the other hand, I am a very firm believer in the desirability of companies keeping as far from the rocks as possible, and consider that the companies should of their own free will set aside far higher reserves than the Government calls for. The strength of a company consists in the excess which it has on hand over and above the legal standard of solvency. Do not, however, let us mix up the two essentially distinct things, the reserve necessary for solvency and the reserve expedient from the standpoint of strengthening our companies.

MR. C. C. HALL: It seems to me that the plan advocated by Mr. Dawson is open to this criticism, that it seems to be devised for the purpose of evading or concealing the effect of the greatly increased expenditure for obtaining new business that has arisen of late years, by offsetting an anticipated saving in mortality. The business of life insurance has grown to enormous proportions and it has been made a success without recourse to any such expedients. The companies that have grown to great size did so by maintaining the standard of reserve on first and second-year policies as well as on those of longer duration.

Now, if coming face to face with the fact that the cost of obtaining new business has greatly increased, we seek to adopt a method of reserve which will dodge the necessity of recognizing that expense, I think we tread upon dangerous ground. Furthermore, the fixing

of the reserve by means of a distinction between different classes of policies, which is not recognized in calculating the premiums charged for those policies and the provision made for normal cost of insurance, has, in my judgment a dangerous tendency. A reserve, as I understand it, has no meaning except as a reserve held by the company for the aggregate insurance in force. For the purpose of calculation we deal with policies separately by ages at issue, by number of premiums paid, by different plans. That is the mode of construction, just as a house is built by placing brick upon brick, but a pile of bricks does not constitute a house, and great confusion of mind has been caused by the language often used, offering as a surrender value "the entire reserve upon the policy," meaning the undistributed share of the company's reserve. The phraseology is unfortunate and misleading.

MR. SANDERSON: A mortality table is, I take it, simply a sort of yard stick by which, in combination with the rate of interest, we measure the policy liabilities of a company. What is the most accurate yardstick or mortality table that can be obtained and practically applied? Previous speakers have referred to different tables. Mr. Macaulay has urged the adoption of an ultimate table (such as $H^{M(5)}$) as proper for valuations. It seems to me that theoretically the true table to use would be select tables, and we would not be justified in using an ultimate table unless the reserves brought out by such table approximated to those brought out by a valuation by select tables. The justification for the use of the two tables H^{M} and $H^{M(5)}$ in a valuation is that the net result is approximately the same as a valuation by select tables. No sufficient reason has yet been given why an ultimate table alone is the proper measuring stick.

MR. MCCLINTOCK: The question of making premiums and reserves by select tables is in this country absolutely theoretical. No company is going to do it, no state is going to pass a law to make reserves according to such a system; and if any company were to do it, of its own accord, it would speedily find that the pressure upon its resources would be heavier than it would be under any present known system, by reason of the heavy reserves not being compensated by gains on mortality during the early years.

It might be added also, technically speaking, that the gains from lapses during the early years would not be so considerable as they are under the present system; but without going into details, it is enough to say that no one who is responsible for guiding a company would be able to make any American company adopt such a system or make any American legislature approve of it.

Now, practically speaking, we have the American table in possession. The American table is employed as an ultimate table for all business after five years by American companies. I say this notwithstanding the truth of what Mr. Van Cise said, that in occasional cases the mortality in some quarters may exceed the American table. There

may possibly be some little local companies for which the mortality exceeds the American table, but every good life insurance company doing business in New York is doing a business which is not going to show a mortality as high as the American table at any time during the continuance of that business. The American table is for practical purposes an ultimate table, and the premiums and reserves are by law required to be valued according to that table.

And then there is something more, something which has not been referred to this morning, something which dominates the whole question, and that that is possibly a majority, certainly a large proportion of the working companies of this country—I do not say in extent of business but in number of companies—are working on a plan which we know as the one year term plan. The number of those companies is going to increase. It will not be many years, if the present conditions continue, before a very considerable proportion of the business of this country is done on that plan; and the question is: What are we going to do about it? That is the question; and discussions of what might scientifically be a good thing for companies starting in Japan do not apply to the situation in this country. What we must discuss is, what we are going to do under the present condition of the laws, which permit not only one year term reserves but two year term reserves and three year term reserves and five year term reserves, and any other term reserves that you choose to put in.

I say the law permits it. As most of you know, I have repeatedly denied that the law does permit it, but I seem to have been overruled by the judges. Either we ought to have a law regulating reserves, or we ought to have no law regulating reserves. That sounds like a platitude, but I would like to have everybody here put it straight to his own mind and ask himself which of those two views he holds. We ought to have a law regulating reserves, or we ought to have no law. Let us say that most of us agree that we ought to have a law. Personally I do not think so, but let us say that we ought to have a law. If we have a law, let us have a law which is just.

The difficulty of legalizing the one-year term valuation for companies which desire it (regardless of the fiction by which what is really a life policy pretends to be a term policy) lies largely in the difficulty of applying that system equitably to other than ordinary life policies.

Now, this proposition of Mr. Dawson's is a practical proposition effecting that very object. It is, as Mr. Wells has stated, intrinsically unobjectionable, assuming, of course, as he said, that the mortality table used after five years is sufficient for the mortality at that time, and the percentages used before the end of five years sufficient to cover the mortality meanwhile. I think we can none of us deny that a reserve calculated on that basis is at any rate sufficient to meet the risks, provided the premiums are large enough.

If the members of this Society, not acting as a Society, individually choose to look the matter in the face, and to recommend the passage of laws permitting to companies desiring it a valuation on this new

system, and forbidding the valuation of one year term and two year term policies in accordance with their apparent terms, but requiring them to be valued according to the intrinsic merits of the case—if they will use their influence in this direction the inevitable effect will be, sooner or later, for the Commissioners and legislators to perceive that they have now a system which is no system, a system which leads to chaos, and that this other system is carefully guarded and in five years gives the entire full reserves that the old system does, and during that period gives nothing more to those companies except the gains from mortality, which is only anticipating a little the gains which they are bound to get in any case.

Mr. Nichols: It will probably be remembered by a number of the members of this Society that I was some years ago the chief sinner, in that when the Vermont Commissioners suggested that the Society should take up this question as to valuation, I ventured to discuss it from the legal side. I did not undertake to say what the law should be, but merely what the law was, and the law turned out to be as I predicted. We are up against not only by a mere theory but also by a situation in another respect, which was recognized by the Supreme Court of Vermont in its decision; the moment you undertake to insist that the net valuations shall be made along the old conservative lines, the young companies come forward and say, "that drives us out of business, and establishes a monopoly." The cry of monopoly, if raised throughout this country would be, I presume, too strong to be overcome; and I am in doubt whether it would be possible to secure from the legislators, even if it were recognized as advisable, a law which would prevent something like the present system of Preliminary Term valuation, or some modification which would permit the public to freely start new companies.

I think Mr. McClintock is right in that either the law should be modified so as to deal with the situation consistently, or there should be no law at all with regard to valuations.

Mr. Fackler: There are some things in what Mr. McClintock said that I think should be touched upon. I am surprised to hear him speak of the Preliminary Term of one year as being equivalent from a legal point of view to one of two or three or four years. Now, that idea has been expressed a number of times but never by anyone of Mr. McClintock's standing and authority, and I think the idea should be met by a counterstatement in this meeting.

If I understand the theory of the Preliminary Term valuation it is to charge a premium during the first year contract at a higher rate, the idea being that the actual policy goes into effect at the end of the first year and the premium is calculated from the end of that first year. If a company does have the Preliminary Term plan, it cannot have a complete set of tables of all kinds of policies with a Preliminary Term extending over two years, and Mr. McClintock must know that.

Take a ten-year endowment policy; you cannot have a working premium computed with only eight years' premiums to make up the endowment. It is difficult to arrange to have the premium accumulate only nine years. The Preliminary Term is practically, by the necessities of the case, limited to one year.

Now, there are really very great questions of, you might say, public policy and public economy involved. We know that the net valuation situation is in one way a failure and that companies that would be in existence to-day have been strangled by the operation of the net valuation law.

Take the case of the Atlantic Mutual, which was thrown into the hands of a receiver, or at least, to speak more accurately, which was brought up before the Court on the questions as to whether it was insolvent, and after a careful examination they found it was perfectly solvent, but the Judge said, "This litigation has done so much harm to the company that we might as well go ahead and appoint a receiver." So they appointed a receiver and wound it up, as has been the case with a number of companies, more or less.

We all know that the true system is a carefully modified gross valuation, in which not alone the saving in the earlier year's mortality but also the unequal distribution of the expenses should be considered. The latter really is a matter of far greater importance, as Mr. Macaulay says, than the matter now brought up. We know that the expenses must be larger in the first two years than afterwards, and the system that does not recognize that cannot be called perfect. We all know that net valuation is a mere rule of thumb, and when it was adopted the first year's expenses were not comparable to what they are now, as companies were paying ten per cent. first year's commissions and five per cent. renewals, sometimes less than that. There is no comparison between the condition of affairs now and what it was when the net valuation system was in order.

The facts are these; we are under an unjust law, an unjust rule of thumb, so to say, and as always happens when such a condition of affairs exists, there is found some way of getting around it; it has been discovered that there are loopholes in the law, and the companies are getting around it.

While the companies having "Preliminary Term Policies" cannot give the same surrender values as the older companies do, still they give good ones after the completion of the first year.

It has been contended by some that this "Preliminary Term System" is a fraud on the public because it enables them to expend all the first year's premium. Is not that practically what all the companies do? and don't they borrow from the old members to make good the reserves?

The Preliminary Term companies meet the exact facts of the case; they do not dodge them; and so far as I am acquainted with their management they are paying in some cases less, and in other cases no

more first year's commissions than the other companies they are competing with. These things must all be kept in mind in arriving at a judgment upon the case as a whole.

MR. MCCLINTOCK: Is Mr. Fackler acquainted with any company, by hearsay or otherwise, that works on the two year term plan?

MR. FACKLER: I cannot say that I am.

MR. MCCLINTOCK: I have heard it said that there are two companies in this country working on the two year term plan.

MR. FACKLER: I have heard some report to that effect.

MR. MCCLINTOCK: Inasmuch as I have heard it said that companies were working on the two year term plan, and as I have heard Commissioners say that there is no reason in law why they should not have five year term plans, I think my remarks are pertinent to the subject. There is no legal limit at present to the extension of this term system.

I agree with Mr. Fackler in almost everything that he says—everything, I think, except his doubts on the legal similarity of terms of one year and of two or more years.

One year term companies, according to Dr. Sprague's original system, begin the reserve the second year, on the ordinary life plan and other policies have correspondingly reduced reserves at the end of the first year, because Dr. Sprague's original proposition referred only to the ordinary life policy and the principle of it would require some reserve on higher premium policies in order not to introduce discrimination between policy holders of the same age. The companies which operate on Dr. Sprague's original proposition have my entire sympathy and support.

Nothing can be said too strong to corroborate Mr. Fackler's statement that the net valuation law has in the past strangled many small companies which might have gone on and become good companies. Nothing can be more true than that the older companies to-day would not be in existence if the present net valuation law had been stringently enforced during the first few years of their existence.

The use of this system of Dr. Sprague was advocated by me five years ago provided it could be made legal. I was asked afterwards whether I thought it was legal; at the time I thought not. I still think so notwithstanding that the courts have overruled me. If it could be made legal, and if the reduction in reserves on other forms could be assimilated to the reduction on the ordinary life plan, I should have nothing but praise for that system.

As regards the publication of the results of the valuation by the Commissioners I would say that if the one year term is legal, or if Mr. Dawson's system is made legal, in any state, the Commissioner in that state ought to be required, not according to his judgment or will but by law, to make a separate statement about the companies

which carry a full reserve on the net system and those smaller and younger companies which adopt either the one year term system or this system of Mr. Dawson's.

The necessity of some law modifying the stringent net valuation system for the benefit of new companies is, I think, apparent to everyone who has investigated the subject. Now, if we come to compare the one year term system with Mr. Dawson's system, we must give credit to the latter for being the more conservative, because it applies in a mathematical and intelligible way to every kind of policy and after five years you have your old full reserve which you do not have under the other system.

MR. CRAIG: I know of contracts issued by two companies providing for a two year preliminary term, and there is one thought in connection with this general question which has not been touched upon at all, but which it seems to me will in years to come be of vital importance.

Those gentlemen who have been in the business many years recall the fact that from about 1865 to 1870 there seemed to be a mania for the establishment of life insurance companies. Nearly all of the companies that were established during those years have ceased to exist. As a matter of fact, in the State of New York alone during the years 1866, 1867 and 1868, there were sixteen life insurance companies organized, only one of which is in existence at the present time. There seems to be a spirit all over the country to-day to organize life insurance companies and it would seem as if they can only be organized and operated under this one year preliminary term plan, and the thought in my mind is that with the ambition which apparently controls most executive officers in life insurance companies, to make the largest showing they can with the money at their disposal, there will be the tendency to spend every dollar available for securing additional business; and in a few years, notwithstanding the permission to use entire first year's premiums for expenses without any liabilities save the death losses which occur during the year—there will be a repetition of history, namely the winding up of life insurance companies; and if this expectation is fulfilled, the principal companies, the companies that have won the confidence of the public, are more sensitive to-day to a failure of other companies than they were twenty-five or thirty years ago.

Two, three, or four failures in life insurance to-day would have as much effect upon the business of perfectly solvent and strong companies to-day as it had in the early years, and the question will arise, are you gentlemen prepared to receive the applications to reinsure the risks of those companies, and if so, on what conditions? Will you take them as they are; if so, and you do business in those states that do not acknowledge the one year term principle, then you must put up the full reserve.

There was one remark that Mr. Fackler made that I was very glad to hear, because it is so different from the popular idea. He stated

that the companies to-day borrow money from the existing policyholders to make up the deficit on the first year's business. That is so. They borrow it. They do not take it; they do not appropriate it. It is not at the expense of the existing members that the new business is secured. The money is simply borrowed, and it is paid back, every dollar of it, when the fund is made good. This, however, does not justify any scheme which has for its object a reduction of the initial reserve and thus does away with the necessity of securing funds by borrowing or otherwise to cover the ordinary reserve.

MR. HANN: Why should they not? What is it used for but to bring in more members in order that your future will be more profitable? I say that money put out in that way is well expended and justifiable.

Mr. SHEPPARD: Referring to the American table again, I wish to call attention to this point. Mr. Fackler stated, if I remember correctly, that the American table can be considered as an ultimate table because it is based on experience in which the first five years of insurance were omitted, but when we consider that the Council of the Institute of Actuaries decided that after five years selection had practically worn off in the case of the earlier experience (giving the $H^{M(5)}$ as a result), while in the more recent experience it has been necessary to increase that period to ten years, and when we consider that this may be due to increased efficiency of the medical examination, it seems probable that the same has happened in this country and that it would be necessary to exclude more than five years of insurance in order to get an ultimate table. If my argument is correct, the American table cannot be considered as an ultimate table.

MR. MILLER: The Mutual Benefit is in the habit, whenever requested so to do, of issuing policies on January 1st, I will say, but making the regular premiums payable July 1st, or October 1st, in order to meet the express wish of a man to have his regular premiums fall due at a season of the year when it is convenient for him to pay. We issue a good many policies of that kind. It is true that we draw the policy so that the premium for the first six or eight months is expressly a term premium, and at a very low term rate, but we provide that at the option of the insured, when July or October comes, he may continue that policy. Does the premium for the first six months cease to be a term premium merely because the party has the right to begin paying the regular premiums on July 1st, at the regular rate for his age on July 1st? The difference, it seems to me, between our policy and the usual Preliminary Term policy which we have been talking about lies simply in the amount that we charge the insured.

MR. WELLS: We do the same thing. We call it temporary insurance, but not a term policy. But take a policy which may be continued at the option of the insured with an increasing premium, that

increasing premium being the net term premium loaded by a uniform percentage and it makes no difference whether you call those policies life or term policies, the reserve will be precisely the same. It makes all the difference in the world whether you make the insured pay a uniform annual premium or whether you make him pay an increasing premium.

There are a number of companies which issue renewable term policies, but really, if they may be renewed at the option of the insured, they are life policies, and if it makes a larger reserve they should be valued as such. But if the premium be the term premium with a uniform percentage loading, you will get the same reserve precisely, whether you value them as life or term policies.

MR. FACKLER: The principle of the net valuation law does not take into consideration any account of premium at all.

MR. WELLS: The net valuation law may be a humbug.

MR. MACAULAY: I would not like the statement to pass without comment that the true basis for the valuation of policies is some select table. I think that is absolutely incorrect. Such a basis simply ignores the gains in mortality, due to initial medical selection, and says that the companies should not be allowed to take any advantage of this in the early years of the policies. That is unjust, and I do not think we should take that method of valuation into our consideration at all as being a correct one or one by which others may be judged. If we ignore the fact of the initial expense we must also ignore the medical gains. The only legal standard that then remains is an ultimate table. At the same time, this is a side issue, and the point which has been discussed this morning, as to the best way of meeting the present requirements of the actual situation is worthy of our most serious consideration, is of far greater importance.

MR. VAN CISE: I suggest that the matter might be cured by some such law as we have in New York; that no contract should be loaded over fifty per cent—that the preliminary charge for loading should not be more than fifty per cent of the net premium.

METHOD OF FINDING A SINGLE LIFE EQUIVALENT TO SEVERAL JOINT LIVES BY ADJUSTING TABLES FOLLOWING MAKEHAM'S LAW—DOUGLAS H. ROSE.

PAGE 421, VOL. VII.

WRITTEN DISCUSSION.

MR. GAYLORD: Mr. Rose's paper in *Transactions* for May 1903, as well as his former paper in *Transactions* October 1902, showing the two methods of obtaining annuity and insurance values by substituting one life for two or more joint lives, is of a good deal of interest, though mainly so to the student, as showing the many convenient characteristics of a mortality table which follows Makeham's Law. For practical use, however, in calculations of office premiums and reserves as the basis of surrender values, at least for two or three joint lives, there is room for some doubt as to their advantage over the method of using equal joint lives, especially in view of the fact that at the present time not only commutation columns but also tables of reserves and premiums under the latter method have already been published on the Makehamized American table, which cover the great majority of cases that will generally arise in practice, and in favor of which there is the further advantage of extremely simple and direct calculations. In fact, so far as office premiums are concerned, it is only necessary to furnish an agent with a table of premiums for equal ages and a subsidiary table of the additions to the younger age, and he can directly calculate the required premiums himself.

The methods which Mr. Rose outlines, however, are not only interesting but would also be very useful in checking values otherwise obtained, and in calculating values on tables following Makeham's Law under which joint life columns have not been prepared, or on four or more joint lives on the Makehamized American.

The later paper treats of the adjustment through a change in the rate of mortality instead of in the rate of interest, and this would seem to be the more consistent method, although the results in each case are of course the same. The so-called adjusted mortality method after some additional labor at the outset, gives the C_x and M_x columns directly with smaller chance of error, and would doubtless require less labor ultimately, though when used as an approximation, the adjusted interest method can be applied in obtaining annuity values from annuity tables at the more regular rates of interest by interpolating for an annuity at the adjusted rate. In this connection it may, however, be noted that, as the addition for each life, as Mr. Rose shows, is approximately .0080, we would have to have annuity tables at rates at least as high as 6% in order to interpolate and obtain values on four or more lives.

MR. MOIR: The use of Makeham's Law as applied to the graduation of mortality tables is such an important subject that any phase of the question submitted to us should have careful consideration,

and the authors deserve our hearty thanks. It is a comparatively new subject in its practical application, although the theory was propounded so long ago as 1860. So much practical use is now made of it that no practicing actuary can afford to neglect the study of the system. Whenever problems involving two or more joint lives come up for solution this law proves of immense value; and it must always be remembered that the results thereby obtained are not approximations but are rigidly correct, depending only upon the accuracy with which the arithmetical work is conducted.

We have to thank Mr. Rose, therefore, for again placing this subject before us, and for the interesting way in which he reminds us that instead of modifying the rate of interest the modification may be made on the force of mortality if it be designed to substitute a single life for two or more joint lives. However, I look upon this side of the question as being interesting in theory but not in practice, because I favor using the same number of lives of equal ages instead of substituting a single life at a modified rate of interest, or with a modified force of mortality.

The question of tabulation is the same in either case ; the formation of the necessary tables is, if anything, more easy when lives of equal ages are involved; the force of mortality can be conveniently used for obtaining the equivalent ages (*Text-book*, page 213); and there is room in this direction for still further developments of Makeham's Law, in the case of a mortality experience which does not follow his first development. For example, I cannot see, on a brief consideration of the question, how the system of using the changed interest rate could have been applied to select tables along the lines which Mr. Hardy recently explained for the graduation of the new British select experience.

Under the system suggested by Mr. Rose it is necessary to go back to the original table of mortality, modify the values of l_x and d_x, and tabulate all values from that stage onward. To this extent more tabulation is necessary than under either the "modified interest" method or the "equal age" method.

There is one criticism which I should make regarding the figures submitted by Mr. Rose and the attitude he assumes towards them. He gives, on page 424, two sets of figures deduced in different ways; the first considering the adjustment as in the interest rate, and the second considering it as in the force of mortality. He shows how close these figures work out to one another, but they are mathematical identities and therefore must be exactly the same if the calculations are properly made. The difference in the last decimal place is caused solely by the number of decimals employed—if greater accuracy were introduced into the arithmetical work, the difference in the third decimal place would disappear.

In like manner, Mr. Rose states, foot of page 425, "the writer has sometimes used one method as a check upon values obtained by the other, thus bringing both into service." Here again the two methods

are mathematically identical, and there is therefore no need to employ the one as a check upon the other. All that is necessary is to make the calculation correctly by one method and the result is correct, just as certainly as multiplying $6 \times 5 = 30$ gives the same as multiplying $2 \times 3 \times 5 = 30$.

AN INVESTIGATION IN THE MORTALITY RATES OF THE CITY OF NEW YORK

JOHN F. ROCHE.

PAGE 426, VOL. VII.

WRITTEN DISCUSSION.

MR. HANN: I think that the idea of taking New York City as a measure of the national experience is a very glowing one, but I wish to point out how we have been educated up in the preparation of statistics. When Adam Smith was preparing his famous work on the "Wealth of Nations," Great Britain was judging what the population was. The first census we presented was in 1809 and I would say the two essential considerations for the compilation of such statistics would be uniformity in the material called for and simultaneousness in the collection of it.

Now, it may not be known to every one, but it is to most of you, that the political condition of this country was such as to compel an early recognition of the necessity of having a census, and therefore, in the Constitution there is a provision for a census to be taken every ten years. Between the years 1787 and 1790 estimates were made in order to have the proper representation in Congress; but when the facts were presented it led to much dissatisfaction because it did not come up to the expectation of the people in regard to the numbers. They thought it would be much larger.

At the recent Actuarial Congress we had the representative of the Census Bureau present. He met us half way, and said that if we had any suggestions to make hereafter, he would be only too glad to have them made. Speaking first of the material wanted, we have always got to bear in mind that the census began with an attempt to number the people and then it passed on gradually to the introduction of other elements which made it so complex that it was very difficult to find men competent to collect the material.

Now, if you could take a bird's-eye view of the stream of life passing through space and realize how impossible it is to get anything like accurate figures, where we are passing and repassing each other in numerous ways, you can form some faint idea of what the Census Bureau had to contend with under past conditions.

Then again, state rights have stood in the way of having a national system in regard to marriages and deaths; and those things all had to be considered, and we find that in 1870, I think it was, Mr. Garfield was appointed Chairman of a Committee to see what could be done to remedy certain difficulties in the schedules for the collection of material for the Census, and that he recommended to the House that

they reduce the time of enumeration to one month, and that then it was laid before the House of Representatives and approved by them, but was thrown out in the Senate; so that the Census of even 1870 had to be taken by precisely the same old, unwieldy plan of 1850.

Now, I think we can in these meetings recognize the great difficulty that the Government had in educating the people to this fact that the more you introduce into the papers, in the way of questions to be answered, the more you perplex, the more you irritate, the more you drive off people from giving you correct returns; but if, at the same time, we are going to have anything like a correct enumeration of the people in the whole country or of any city, it is plain that it has got to be taken on the same day. You say this is impracticable. It is not impracticable. You have for instance, your registrars or you have that which corresponds to them, all over this country, and we could have the enumerators obliged to deliver their papers with instructions to the holders, or whoever they may be, to fill them up at a given time. It is done so in Great Britain, and they have about ten days to go around, not having more than two hundred houses to attend to; they deliver their papers containing about seven or eight simple questions, and they call, in accordance with the agreement, for those papers on the morning succeeding the day on which the Census was to be made.

We are much indebted to Mr. Roche for the interesting paper he has prepared, and I think his taking up the City of New York as a type of the national statistics is not a bad plan.

Take a vast amount of insurance, passing through a generation of time of say forty years, and let the expected mortality in the first year be represented by x, and in the first five years you will find that you have to add to that about twenty per cent. Take the State of New York and you will have to reduce the mortality about eleven per cent., and in the first five years it is much less. Taking it after the first five years, you have the experience of the whole of the United States, and that of New York State almost alike, and for the whole of that time it will be in very close confirmation of what Mr. Miller has stated this morning.

Mr. Gore: There is one little fact concerning the mortality of large cities which sometimes has considerable weight. Where a large city is the only large city in all of the adjacent area, the inhabitants of a great many regions come into the city hospitals, as we all know.

Recently, in looking into the mortality of the city of New Orleans, I found they kept records of the patients in the city hospital the death of those patients reported as deaths in the city of New Orleans increased the death rate in the city, especially at the middle and older ages, very materially. In a city of the size of New York that would not be true, because the neighboring cities have good hospitals, but it will affect the city mortality no doubt. I do not know whether anyone knows how much, or not.

RESERVES ON INSURANCES UNDER WHICH A SUPER-NORMAL MORTALITY IS EXPECTED—ARTHUR HUNTER.
PAGE 446, VOL. VII.

WRITTEN DISCUSSION.

MR. HENDERSON: This is a contribution to the solution of the problem of the effect on reserves of variation in the mortality assumed as a basis of the calculation. The direct method being adopted of calculating the reserves on the various bases and tabulating the differences. A good deal of light can be thrown on the reasons for these variations by investigating the problem taken up in the Institute Text Book, namely, what laws of mortality will give the same terminal reserves. It is evident that if the terminal reserves are throughout the same on any two tables, the difference in net premiums with one year's interest must be equal in each year to the difference in the cost of insurance by the two tables, or to the difference in the rates of mortality multiplied by the amount at risk (s—v). The difference in net premiums is a constant for any policy during the premium paying period, so that the difference in rates of mortality must vary inversely as the amount at risk during that period. Where no premiums are payable, as in the case of a limited payment policy after the payments have all been made, there can can be no difference of net premiums and consequently none in rates of mortality. In the case of regular endowment policies, the conditions above stated would require an indefinitely great difference in the rates of mortality in the last year of the endowment period—a condition impossible of fulfilment. In fact, it is readily seen that the larger the net premiums the smaller the terminal reserve in the second last year of the endowment period. Accordingly a table showing higher rates of mortality necessarily produces somewhat lower reserves during the latter part of the endowment period just as it necessarily produces higher reserves on limited payment policies after the expiration of the premium period and also towards the end of that period. The effect of any given table of mortality can be roughly measured by constructing the table of mortality which would give the same net premiums as does the given table and the same terminal reserves as the table adopted as a basis of comparison. A comparison of this hypothetical table with the one in question will show which gives the higher reserves from the consideration that if up to a certain point one table shows a higher rate of mortality than another, the net premiums in the two cases being the same, the former will show a lower reserve at that point.

On the other hand, the table showing the higher rate of mortality for all ages after a given point will also show the higher reserves after that point, the net premium being still supposed to be the same. The results arrived at above for endowment reserves indicate the reason for the peculiar results shown in the paper for reserves on endowment policies with a lien imposed. The explanation is that the rate of mortality assumed is so high that the net premium on the lien policy on

that assumption is greater than the net premium for a regular policy on the assumption of normal mortality, although the gross premium charged is the same in the two cases.

The only conclusion arrived at in the paper to which I am disposed to take serious objection is the fifth, namely, that the case of a table showing an increasing percentage of the American experience rate of mortality is not worthy of consideration. The mortality at an age older than the given one by a fixed number of years shows a law of this nature, and it is, of course, evident that the resulting reserves will be higher than those based on a level percentage of extra mortality which gives the same net premium. Valuation by the rated up age has the merit of elasticity, being readily adapted to varying degrees of extra mortality without prohibitive labor and, in my judgment, furnishes a more satisfactory method of erring on the safe side than does the assumption of a uniform extra mortality for all cases which is supposed to be higher than the average extra mortality covered by the rating. This is entirely apart from the debatable question whether there are as a matter of fact any classes which do show increasing extra mortality.

THE SPECIALIZED MORTALITY INVESTIGATION.

Written Discussion.

Mr. Hunter: Let me state in the first place that this discussion has been prepared at the request of President Pierson, and that there is no intention to criticise the work of the Committee, which is a model of energy, ingenuity and good judgment. My hope is to provoke a discussion on the subject which will result in a reasonable view being taken of the deductions that may be drawn from the Specialized Mortality Investigation.

Mr. Weeks has brought out the fact that the expected deaths by the standard tables adopted by the Committee were 140,851, while the actual deaths were 140,622,—a ratio of actual to expected deaths of 99.84%. It appears to me, therefore, that either of two conclusions may be drawn from that fact,— either the standard tables show too heavy a mortality, or else the lives in the experience are not really under-average or impaired risks. As Mr. Weeks has taken up the first of these conclusions, I shall only deal with the second conclusion.

When we are called upon as experts to criticise statistical work dealing with population or railroad returns, we examine the data upon which they are based to determine what they actually are, and whether the name under which they are classified truly represents them. For example, if we found that the mortality in England among musicians and music teachers was 127% of the standard for occupied males, we should examine the data, and when we discovered that the data included itinerant musicians and organ grinders, we should hesitate to base conclusions on other data in the book without first making a

rigid examination of the data comprising each class. Let us therefore proceed to examine the data of the Specialized Mortality Investigation in the same spirit as we would the Population Returns to which I have referred. I shall consider only a few typical classes, as time does not permit a discussion of all the classes.

In the first place, let us take Class 7 (Colored or Negroes). The ratio of 108% undoubtedly represents the mortality on a specially selected body of colored people. It does not truly show the mortality of the Negro race as compared with the white race *under similar conditions.* The data which were presented by me at the Fourth International Congress contained statistics prepared by Mr. Gore which prove conclusively that the mortality of the Negro as compared with the White under like conditions is about 140% at ages above 20.

Let us now turn to Class 10, consisting of policemen, sheriffs, constables and marshals. The composition of this group is very heterogeneous, as it contains sheriffs from large cities who have no hazard from occupation, and sheriffs in certain parts of this country where the hazard from occupation is very high. The mortality in this group, therefore, depends entirely upon the relative proportion of sheriffs, marshals, etc., in non-hazardous positions to those in hazardous localities.

The classes among the so-called hazardous occupations which to my mind are most likely to lead to wrong deductions, are Nos. 13, 16 and 18. These comprise people exposed to electricity, workers in iron and steel at high temperatures, and potters. The percentage of actual to expected mortality is 81%, 90%, and 78% respectively. It is not likely to be argued that these classes of people are very much better than normal "accepted" lives, and one conclusion only can be maintained, namely, that the very best of the people in these occupations were taken, This can be readily understood if we inquire into the method of business employed by most of the companies. Very few of the insurance companies in the past made provision for those who were exposed to a high voltage in electricity, unless under their sub-standard plans, and as sub-standard cases were excluded from the investigation, it follows that the men included in the group were probably mainly electrical experts and telephone people who are not much exposed to the hazard of electrity. It is also probable that the workers in iron and steel at high temperatures are principally the foremen and other highly skilled mechanics who incur slight danger. With regard to potters, the work is so diversified that in the higher grades there is no extra hazard, while the so-called potters who do work in flint and spar mills are probably as hazardous risks as any other set of men in this country. It may be interesting to mention that the mortality among all grades of potters in England was double the standard for all occupied males.

Groups 15 and 17 comprise the steel grinders and glass workers, the ratio of mortality being 120% and 113% respectively. There is no

doubt that the steel grinders and glass workers were a specially selected body of men, and that 120% and 113% respectively does not show the average mortality among people *of all grades* in these occupations. The relationship to the standard for occupied males in the British experience was 159% for cutters, 190% for file makers, and 156% for glass workers.

The house painters, printers, tailors and butchers appear under Nos. 19, 20, 21 and 22. The mortality in all these classes is below the normal standard, which leads us to beleive that the majority of the people insured in these classes are the employers and not the employes. For example, the tailors are unquestionably the merchant tailors and not the bench workers. The statistics prove that the people whom the companies insured under each of these classifications were a profitable body of lives, but it does not follow that the door should be open to all applicants coming under the generic titles of tailor, house painter, printer and butcher.

In classes 31 and 32 the data do not seem homogenous. We have brewers and their employes, and distillers and their employes. The combination of brewers and their employes is less homogenous than a combination of clergymen and day laborers. The head of the firm in a large brewery who is engaged solely in a managerial capacity and is an abstainer is surely a very distinct risk from the vatman who is allowed thirty glasses of beer a day and never fails to take advantage of his allowance. The mortality among a group of brewers and their employes would therefore depend entirely upon the relative numbers of brewers and their employes. It seems probable that the mortality among these people ranges from 100% to 250%, depending upon the nature of their relative consumption of alcohol. The same general argument holds good regarding the distillers and their employes. Therefore we cannot infer that the manufacture of beer is more hazardous than the manufacture of whiskey because the mortality among distillers and their employes was 107%, and that among brewers and their employes 135%. It may be that those dealing with whiskey are less hazardous risks than those handling beer, but the difference in mortality in the two classes may merely be due to the proportion of policy-holders in a managerial capacity among the "distillers and their employes" being greater than among "brewers and their employes."

The pilots, seamen and fishermen (Classes 41 and 42) show a surprisingly low mortality. It is doubtless due to the fact that a large proportion of the pilots were men whose duties did not take them outside of harbors, and that the fishermen were largely comprised of shore fishermen. It is a well known fact that deep sea fishermen are almost invariably declined by the insurance companies. From recent statistics which have come into my hands, it appears that the mortality from accidents on the Newfoundland Banks is from 15 to 20 deaths per annum per thousand, in addition to the normal mortality.

We now come to the groups in which there was some defect in the physical condition or personal history, and we must bear in mind in

dealing with these groups that there was no sub-standard business included in the data. It was the custom to decline or treat as sub-standard any case where a consistently intermittent or an irregular pulse was found. The low mortality in Class 43 must be supposed to result from the fact that the group was principally composed of persons whose pulse was found to be intermittent or irregular upon one examination, but was regular on other examinations.

The mortality among applicants who have taken a cure for intemperance is surprisingly low and can only be explained on the ground that no cases were taken unless many years had elapsed since the person had taken the cure, and his record from that time had been one of total abstinence. To use the rating of 125% in all cases of people who had taken the cure would be manifestly wrong, as a man who has taken the cure within a year and now drinks moderatey is a vastly worse risk than one who has taken the cure twenty years ago and has been an abstainer since that time.

The same general argument regarding those who have taken a cure for intemperance may be applied to those who have had syphilis. The group unquestionably contains only those who have been carefully treated and who have passed a number of years without any further manifestation of the disease. In my judgment, therefore, the rating of 135% represents the very best of the applicants for insurance, and is the lowest rating which should be applied to persons who have had the disease in question.

If there were in the past one symptom more than another that decided a company in declining a risk, it was an attack of blood-spitting. The mortality of 108% on those who have blood-spitting must, therefore, represent only those in whom blood-spitting was largely accidental. Without an analysis of the cases no one has a right to speak with any authority on the number of years which had elapsed on the average between the attack of blood-spitting and the date of the application. If I may be allowed to give an opinion, I should like to hazard the guess that the group contains few cases in which the blood-spitting occurred less than fifteen years prior to the date of the application, and that it only included people of excellent build who were in first-class physical condition.

The points I have given are in the nature of an expansion and particularization of the warning already given by the Committee; and in conclusion I would like to add that while the actuary who gives careful consideration to the Specialized Mortality Experience can add materially to his professional knowledge, the inexperienced or careless investigator is apt to draw many wrong conclusions from the experience.

Oral Discussion.

Mr. Gore: As a member of the Specialized Mortality Investigation Committee, I should like to emphasize the warning in the Committee's final report that the data do not relate to certain classes

taken from the general population but to certain classes selected from life insurance with the idea in most cases of guarding against some special hazard. For example the comparatively few negroes recorded in Class 7 were no doubt selected with much greater care than is the case with the average insurance risk. The fact that the actual deaths in this class were only eight per cent. more than the expected is of little value unless coupled with the fact of this careful selection.

It will be well to remember that in many of the classes the numbers exposed to risk were so small that the results cannot be considered conclusive. In more than two hundred of the thousand sub-groups the expected or table deaths were less than ten.

A noticeable result of the investigation is, that while the average actual mortality of the young and mature entrants, ages 15 to 42, is one hundred per cent. of that expected, the corresponding per cent. for the elderly and old entrants, ages 43 to 70, is about one hundred and fifteen, the average for all ages being 106. As the standard table adopted by the Committee for the investigation, namely, Farr's Healthy English Table with certain modifications explained in the report, agrees very closely at all ages with all the published data of American companies, it would seem that in the majority of cases, the hazards that are supposed to belong to the various classes are greatest, and therefore most to be guarded against among the elderly and old entrants.

Through the courtesy of Messrs. Miller and Craig, many of us have been furnished with tables of percentages of actual to expected deaths and of actual to table deaths. The percentages in these tables are found by comparing the total number of deaths during the period chosen with the total expected or table deaths as the case may be.

A word of warning may not be out of place concerning the use of these tables. As the business of American companies has grown so rapidly in recent years, the great majority of the policies are in the first few years of insurance. The method just explained gives undue weight therefore, to those years. Where the numbers exposed are great enough, a more satisfactory result is obtained by finding the average of the percentages for each year of insurance.

I have not had time to get these percentages year by year but have had them calculated in groups of five years of insurance for insurance years, 6–30, and compared them with the percentages given on Mr. Miller's table. Of the seventy-five classes recorded during years 6–30, 55 show a higher comparative mortality by the method of averages than by comparing the total actual deaths with the table deaths. In two classes the results are the same by either method, and in the eighteen classes, the latter method gives higher rates. With one exception, however, these eighteen classes are small and the method by averages is not reliable.

In Class 4, born in Germany, for example, with 1,616,000 exposed to risk for one year, the method by averages shows one hundred and

ten per cent. of the table deaths, and the other method one hundred and six.

In Class 12, physicians, with 557,000 exposed the corresponding ratios are 107 and 103.

In Class 22, butchers, 225,000 exposed, 97 and 89.

In Class 26, traveling salesmen, with 269,000 exposed, 97 and 89.

In Class 47, those who have had inflammatory rheumatism once, 103 and 100.

In Class 57, those who have had dyspepsia, 91 and 85.

In Class 62, 235,000 exposed, neither parent dead below 70, 135 and 109.

In Class 63, one parent or both dead below age seventy, 409,000 exposed, 148 and 136.

I have selected only cases where the number exposed to risk was great enough to make the results reliable. In some instances the differences are unimportant, but as a rule, they suffice to show that the method of comparing total actual and table deaths minimizes the effect of an increase in the actual deaths as compared with the table deaths.

Abstract from the Minutes of the Annual Meeting of the Actuarial Society of America, held in New York, N. Y., on Thursday and Friday, May 19th and 20th, 1904.

FIRST SESSION.

Board Room
of
The Metropolitan Life Ins. Co.,

May 19th, 1904, 11:00 A. M.

The meeting was called to order by the President, Mr. Pierson.

MEMBERS PRESENT.

Alsop,	Johnston,	Pierson,
Craig,	Lee,	St. John,
Fackler,	Lewis,	Sanderson,
Gaylord,	Linzmeyer,	Sartelle,
Gore, J. K.	McClintock,	Sheppard,
Graham,	Macaulay,	Smith,
Hall, C. C.	Marshall, E. P.	Strong,
Hall, S. S.	Messenger,	Tatlock,
Hann,	Miller,	Torrey,
Henderson,	Moir,	Van Cise,
Hildebrand,	Nichols,	Weeks,
Hunter, A.	Ogden,	Wells,
Hutcheson,	Papps,	Woodward.
Ireland,	Paterson, J. S.	

ASSOCIATES ATTENDING.

Angell,	Craig,	Hughes,
Bliss,	Davenport,	Kaufman,
Brown,	Dickenson,	McKellar,
Bulkley,	Dow,	Morris,
Buttolph,	Fackler,	Stanley,
Carpenter,	Flynn,	Washburn,
Cathles,	Hammond,	Washburne.
Cole	Homans,	

After the calling of the roll, the President addressed the Society. (See page 60.)

The minutes of the Meeting of May 21st, and 22nd, 1903, were approved as printed.

The Secretary reported that the following named Associates, having successfully passed the examination prescribed by the Council, had been admitted as Members of the Society under date of May 13th, 1904:

M. M. Dawson, Percy C. H. Papps.

The Secretary further reported that the following named candidates, having successfully passed the examination prescribed by the Council, or having passed an equivalent examination of another Society, had been enrolled as Associates of the Society under the dates as stated:

Lawrence M. Cathles,	May 19, 1904
Richard H. Cole,	May 13, 1904
Benedict D. Flynn,	May 13, 1904
H. P. Hammond,	May 13, 1904
C. W. Jackson,	May 13, 1904
John A. McKellar,	May 13, 1904
Abel Travassos,	May 13, 1904

The following statement regarding the changes that had taken place among the Members and Associates of the Society during the past year was then presented by the Secretary:

	Members.	*Associates.*	*Total.*
Existing, May 15, 1903,	123	29	152
Admitted and Enrolled	2	7	9
	125	36	161
Died - - -	3		5
Associates become Members - -	...	2	
Existing, May 19, 1904,	122	34	156

The President announced that he had received telegrams of congratulation on the fifteenth anniversary of the Society from the Association of Belgian Actuaries and from Mr. Holcombe and Mr. Welch. On motion the Secretary was instructed to acknowledge these despatches on behalf of the Society.

Mr. St. John, the Chairman of the Committee on the Fackler Associates' Triennial Prize, presented the report of the Committee, awarding the prize to Mr. Arthur Hunter for his paper entitled *The*

Makehamized American Table of Mortality, published in No. 27 of the *Transactions*.

The Secretary announced that Mr. Fackler had communicated to the Council his intention to renew his offer of the prize and read the following extract from a letter addressed to the Council by Mr. Fackler, under date of March 17th, 1904:

"I would state that, with the approval of the Council, I should like to renew the offer in a slightly changed manner.

"It has seemed that it would be better to separate the award into two prizes, one of $100, and one of $50, in order to encourage a larger number of Associates to present papers, and further (though the intention was to have a triennial award), as the occurrence of the International Congress of Actuaries last year prevented our usual Fall Meeting, and as there had been no definite announcement of the continuance of the award for another term, it would be better to have the prizes, if approved by the Council, apply to all papers presented after the last annual meeting and up to and including the annual meeting in 1907, so that from the time of the announcement of this award there will be three full years for the preparation of papers.

"The competition would, as before, be open to all papers presented while the writers held the rank of Associate between the annual meeting in 1903 and that in 1907."

On motion, duly seconded, the thanks of the Society were extended to Mr. Fackler for the renewal of his offer of this prize.

The following extract from the minutes of the Council was then read for the information of the Members and Associates present:

At the meeting of the Council held on November 5th, 1903, it was decided that in future the reports of the discussions appearing in the *Transactions* shall be printed in two parts; first, under the head of written comments or remarks written for the discussion by members absent and present at the meeting, unless those present prefer to treat their written memoranda as private and have their remarks appear under the heading of oral discussion; second, the oral discussion, consisting of contemporaneous remarks made by those present at the meeting.

The annual report of the Treasurer was read by the Secretary, in the absence of the Treasurer, and on motion, duly seconded, was accepted and ordered placed on file.

At the request of the President, the Secretary then read the obituary notices of the recently deceased Members of the Society, John B. Tennant, James Meikle and Arthur F. Burridge. (See pages 101, 102, 103). At the conclusion of the reading of these notices, Mr. Macaulay and Mr. McClintock addressed the Society, paying their personal tributes to the memory of these Members, just passed away.

The Society then proceeded to the election of officers for the ensuing year and of two members of the Council.

The following were duly elected: President, Israel C. Pierson; 1st Vice-President, Rufus W. Weeks; 2d Vice-President, Daniel H. Wells; Secretary, John Tatlock; Treasurer, Archibald A. Welch. Members of Council, Messrs. James M. Craig and John K. Gore.

The Secretary read the proposed amendment to Article IX of the Constitution which, as duly notified to the Members of the Society under date of December 31st, 1903, read as follows:

To insert after the words "candidate has" in the last sentence of Article IX of the Constitution the following words: "when not a resident of the United States or Canada."

Mr. Fackler moved that the word "while" be substituted for the word "when" and, the substitution being approved by the proposer of the amendment, the motion was adopted. The amendment to the Constitution, with the substitution as indicated, was then unanimously adopted by the Society.

The Society then adjourned until 2:30 p. m.

During the recess the Members and Associates of the Society, present at the meeting, were entertained at luncheon by the officers of the Metropolitan Life Insurance Company.

SECOND SESSION.

THURSDAY, May 19th, 1904, 2:30 P. M.

The Second Vice-President called the Society to order for the continuance of its proceedings.

The Society listened to the reading of the papers presented at this meeting. (See pages 1–59.)

The Society adjourned at 4:45 P. M.

THURSDAY, May 19th, 1904, 7:30 P. M.

Members and Associates of the Society, to the number of fifty-six, met together for the annual dinner at Delmonico's to celebrate the fifteenth anniversary of the founding of the Society.

Toasts were responded to in accordance with the following programme:

TOASTS.

The President of the United States.

The King of Great Britain and Ireland.

The Founding of the Society, - - - - D. Parks Fackler.

The First President of the Society, - - - H. W. St. John.

The Work of the Society during the past fifteen years, T. B. Macaulay.

Promotion to office of Actuary and higher honors, Bloomfield J. Miller.

International Congresses of Actuaries, - - - Emory McClintock.

The Benefits of Association, - - - - - Oscar B. Ireland.

At the conclusion of his address Mr. Ireland presented to the Secretary, on behalf of the American and Canadian Members of the Society, a handsome, old English hall clock, marked with a brass plate containing the following inscription:

To
JOHN TATLOCK
from his fellow members
in the United States and Canada
of the
Actuarial Society of America
in recognition of his services
as Secretary
and especially of his labors in connection
with the
International Congress of Actuaries
of 1903.
May 19th, 1904.

Mr. Tatlock acknowledged this gift from his fellow members in remarks expressive of his great appreciation of the token thus bestowed upon him and of the good feeling evidenced thereby.

THIRD SESSION.

FRIDAY, May 20th, 1904, 10:00 A. M.

The Society was called to order by the President.

The Society then proceeded to the discussion of papers read at the preceding meeting. (See pages 65–94.)

The Society adjourned at 1:00 P. M., and the members were again entertained at luncheon by the officers of the Metropolitan Life Insurance Company.

FOURTH SESSION.

FRIDAY, May 20th, 1904, 2:00 P. M.

The Society was called to order by the President and the discussion of papers was concluded.

Mr. Nichols offered the following resolution which, being duly seconded, was unanimously adopted:

"I move a vote of thanks to our Ex-Presidents who, by their instructive addresses at the banquet, so admirably set forth the work which has been accomplished by the Society during the first fifteen years of its existence."

Mr. Ireland offered the following resolution which, being duly seconded, was unanimously adopted:

"I move to express our very sincere thanks for the renewed, great hospitality of the President and other officers of the Metropolitan Life Insurance Company, to whom we are once more indebted for the use of this room and for the other hospitalities of which we are invited to partake."

The Society adjourned *sine die* at 2:30 P. M.

JOHN TATLOCK,
Secretary,
32 Nassau Street,
New York.

JOHN BELL TENNANT, Secretary and Actuary of the Friends Provident Institution of Bradford, England, died from pneumonia on 19th January, after a very brief illness. He was born on 28th September 1840, and for a short time after leaving school was in the service of a railway company. In 1860, however, he became a clerk in the Friends Provident Institution, and continued in their service until his death. He was appointed Assistant Secretary in 1883, and chief officer with the title of Secretary and Actuary in 1889. He was elected a Fellow of the Institute of Actuaries in 1886, and a member of the Actuarial Society of America in 1903. He was a member of council of the former body at the time of his death. Mr. Tennant was also a member of the recent Congress of Actuaries in New York as a delegate from the Institute of Actuaries, but did not take any prominent part in the proceedings.

Although very closely connected with the Society of Friends both by business and family ties, Mr. Tennant himself attended the Congregational Church. His quiet, unassuming manners and kindly courtesy endeared him to all who had the pleasure of knowing him personally, his attitude being at all times that of a careful and intelligent student, interested especially in anything new to him. After returning home Mr. Tennant, in December last, delivered a lecture regarding his visit to New York and other American cities, which showed that he not only appreciated the hospitality extended towards himself and other foreign actuaries, but also that he applied a diligent mind to the study of questions and improvements which might with advantage be adopted. His own warm and affectionate nature found a response in the hearts of all who knew him, and we mourn his loss therefore as that of a dear friend.

JAMES MEIKLE, F.F.A., F.I.A. We have to record, with great regret, the death of Mr. James Meikle, one of the most interesting members of our profession.

He was born August 18th, 1825, and died on February 5th, 1904, in his 79th year. His business career started in 1839 and he served the Scottish Provident Institution for 63 years, retiring from the Actuaryship of that Company in 1902.

His clearness of mind even at his advanced years may be judged from the fact that in 1902, while Honorary President of the Actuarial Society of Edinburgh, he wrote three papers comprising nearly 100 pages in all, on the following subjects:—

"The construction of a Table of Mortality," "A mode of graduating a Table of Mortality," and "The preparation of a Table of Mortality from observations of various magnitudes."

These papers followed a long series of publications, which are not only of great value to students but are instructive to all Actuaries. One of the principal directions of study and research on the part of Mr. Meikle was in the valuation of Widows' Funds and Friendly Societies. While he was recognized as an able exponent of many subjects, in this particular sphere of work he was an acknowledged leader. One of the first papers he wrote, about 50 years ago, for the Actuarial Society of Edinburgh was entitled "The Rationale of Life Assurance Premiums," and contains many formulas and verbal explanations which now constitute part of the curriculum of all actuarial students, as they have been incorporated in an important section of the Institute Text Book.

It is not within the scope of this notice to enumerate the vast amount of work which Mr. Meikle has done during his life-time. It is sufficient to say that he has presented to the Insurance world more papers than any other Scottish Actuary. He worked for the Faculty of Actuaries in many capacities, including that of President. In a presentation from the Faculty of Actuaries, on his retiring from active business, the following statement was made regarding him:—

"He has not merely left footprints on the field of knowledge through which he toiled as a student; he has very materially assisted in laying out a path through that field by which the students of the present day can pass with greater ease and celerity than was dreamt of fifty years ago. If any of these students are blest with the ability, the zeal, and the vitality that characterize Mr. Meikle, they may not only widen that path, but it is possible they may carry it further on. If they succeed in doing so, I am sure that nothing would be more grateful to their pioneer."

His private life was beyond reproach; his friendship was always valued; and his advice was freely given to all who might require it, especially, perhaps, to the junior members of the profession. Throughout his life-time he enjoyed a wonderful measure of health, and he was in vigorous health until a few days before his death.

His death is not only mourned by those who have known him for many years, but by the many friends he made among the American Actuaries during his attendance at the Fourth International Congress last autumn.

ARTHUR FRANCIS BURRIDGE, Actuary and Secretary, Equity & Law Life Assurance Society, London.

Died at Bexley, Kent, 22nd April, 1904, aged 52 years.

Although many of us only made the acquaintance of Mr. Burridge at the International Congress of Actuaries held here last September, his personal qualities were such that we feel we have lost by his death not only an eminent member of our profession, but also a dear friend.

Mr. Burridge's business career commenced in 1869, when, at the age of seventeen, he entered the service of the Equity & Law Life Assurance Society as junior apprentice. With the exception of five years his business life of thirty-five years was spent in this office. In 1886 he had risen to the position of Assistant Actuary, and in 1888 he left the Equity & Law to accept the post of Actuary of the old Equitable. There he remained until 1893, when he returned to the Equity & Law as its principal officer with the title of Actuary and Secretary. This position he held at the time of his death, which resulted from pneumonia after a week's illness.

In 1881 he became a Fellow of the Institute of Actuaries by examination, and from that time onward he took a keen interest in its welfare and became more and more closely identified with it in the various works which it undertook. In the *Journal of the Institute* will be found the following papers from his pen: "On the Rates of Mortality in Victoria," "On the Construction of Mortality Table from Census Returns by the Graphical Method of Graduation," "On the Rates of Mortality in Australia," "Some Account of the Census, 1801 to 1881," and a historical and statistical paper on what was then a much vexed question, namely "Vaccination." This last paper was read before the Institute just two years ago, and has been reprinted and now enjoys a wide general circulation in Great Britain. In 1889 Mr. Burridge was elected a member of the Council of the Institute: and from 1896 to 1899 he was its Honorary Secretary. During this period, in 1898, the Second International Congress of Actuaries met in London, and he was one of the Secretaries upon whom devolved the task of making the necessary arrangements in connection therewith. He also collaborated with Mr. Ackland as Secretary in London of the recent Institute and Faculty "Joint Committee on Mortality Investigation." In 1889, on relinquishing the post of Honorary Secretary, he was elected a Vice-President of the Institute of Actuaries, a position which he held up to the time of his death. Amongst his professional brethren in London he was most popular, and from 1890 to 1899 was Treasurer of the old Actuaries' Club. On the formation of the present Actuaries' Club—an amalgamation of the old Actuaries' Club and the Institute of Actuaries' Club—he became Treasurer of the new club, and he subsequently was its Chairman for two years. It will thus be seen that he was one of the Institute's most active members in all its branches.

From the many friends Mr. Burridge made during his short visit to America last fall, we can readily understand the place which he held in the hearts of his fellow actuaries in London, and the loss which his death will cause in their ranks.

Mr. Burridge is survived by a wife and family. To them in their deep affliction let us extend the hand of sympathy across the sea.

VOL. VIII. No. 32.

TRANSACTIONS.

OCTOBER 20TH AND 21ST, 1904.

EFFECT OF TOTAL ABSTINENCE ON THE DEATH RATE.

J. G. VAN CISE.

On April 25, 1895, Mr. Emory McClintock read a paper before the Society "ON THE RATES OF DEATH LOSS AMONG TOTAL ABSTAINERS AND OTHERS," in which he gave some very interesting figures derived from the experience of The Mutual Life Insurance Company of New York. This paper is the first one printed in Volume IV of the *Papers and Transactions*, and those who have not seen it, should certainly read it, and to those who have read it, I would advise a careful re-reading, because of the value of the information given. I do not propose at this late date to enter into a discussion of this paper, but will only mention the main facts contained therein.

The mortality experience of The Mutual Life was given up to policy anniversaries in 1889, on the lives of those who, beginning with the year 1875, had stated in their applications that they were total abstainers from the use of alcoholic beverages, and, for a somewhat longer period, the experience was given on the lives of those who had stated that they were moderate drinkers of such beverages. The actual loss on both classes of lives was compared with the expected loss according to certain percentages of the American table, which were fully explained in the paper, and which it is not necessary to repeat here. It is

sufficient to say that the percentages used were very properly designed to represent the maximum expected loss in the various years during which the policies were in force, allowance being made for the element of medical selection.

Upon the lives of those who had declared themselves to be total abstainers, when applying for their policies, the maximum expected loss was $5,455,669, and the actual loss was $4,251,050. Upon those who acknowledged themselves users of alcoholic beverages the maximum expected loss was $9,829,462, and the actual loss was $9,469,407. It was very fortunate that these figures were derived from the experience of The Mutual Life, and were, therefore, of such great magnitude as to free them from any possible accusation of not being large enough to be of any practical value.

The percentage among abstainers of the actual death loss to the expected death loss was 78, while the corresponding percentage among the non-abstainers was 96. Comparing these two percentages, it would appear that the death rate among non-abstainers, as shown by the experience of The Mutual Life, was 23% higher than among abstainers, the difference of 18 being about 23% of 78. If any similar data have been made up, or printed, by any American or Canadian life insurance company, large or small, they have not come to my knowledge; but some statistics of English and Scotch life assurance companies have recently been published, which seem to fully confirm the conclusion which may fairly be deduced from Mr. McClintock's paper as to the value of total abstinence in tending to lessen the rate of mortality among assured lives.

The experience of the Sceptre Life Assurance Society, Ltd., for the twenty years from 1884 to 1903, inclusive, gives the following figures: For abstainers, expected deaths, 1,440; actual deaths 792, being 55% of the expected. Non-abstainers, expected deaths, 2,370; actual deaths, 1,880, or 79% of the expected.

The experience of the Scottish Temperance Life Assurance Company, Ltd., for the twenty years from 1883 to 1902, inclusive, gives the following figures: Abstainers, expected deaths, 936; actual deaths, 420, or 45% of the expected. Non-abstainers, expected deaths, 319; actual deaths, 225, or 71% of the expected.

These figures are given in a little book recently published in England entitled, "ALCOHOLIC BEVERAGES—THE COMPARATIVE MORTALITY OF ABSTAINERS AND NON-ABSTAINERS," and as the book was sent me by Mr. R. P. Hardy, I have no doubt of the correctness of the figures given therein. No information is given as to what table of mortality was used in the computation of the expected deaths, but as the same table was undoubtedly used for both abstainers and non-abstainers, the comparative percentages of actual deaths to expected deaths as between the two classes of lives, would not be materially changed by the use of any other basis for the computation of the expected deaths. In the case of both companies, the difference in the experience as between abstainers and non-abstainers is remarkable, the percentage of the death rate of actual to expected being about 50% higher with non-abstainers than with the abstainers.

The most reliable and comprehensive experience which has ever been published in Great Britain, however, is that of the United Kingdom Temperance and General Provident Institution of London, which was shown in detail in a paper read before the Institute of Actuaries on November 30, 1903, by the Actuary of the Company, Roderick Mackenzie Moore. This paper is entitled "ON THE COMPARATIVE MORTALITY AMONG ASSURED LIVES OF ABSTAINERS AND NON-ABSTAINERS FROM ALCOHOLIC BEVERAGES," and it is certainly interesting reading not only for all actuaries, but also for all others who are interested in vital statistics and social problems.

I shall not undertake to go into the figures in detail, which are given in this paper, but shall merely endeavor to present a summary of some of the facts and figures contained therein. The company in question has always kept very carefully, the records of its policies in two sections—total abstainers being included in what is called the Temperance Section, and non-abstainers in what is called the General Section. Policies on the lives of abstainers have been issued since the year 1841, and on the lives of non-abstainers since the year 1849. The figures given in the experience include the entire experience of the office to the end of the year 1901, thus covering over sixty years, in the case of abstainers, and over fifty years, in the case of non-abstainers. The same premiums have always been charged for both sections, the total abstainers receiving any advantage arising

from their superior vitality in the form of additional bonus or dividend.

The Temperance section is composed entirely of total abstainers and the General Section, of moderate drinkers, selected, it is claimed, with the same care which is exercised by other companies in the selection of their risks. The habits of the assured are looked after annually, and transfers are made from one section to another, in accordance with the changed habits of policy-holders as to drink; but as these transfers have been comparatively small in number, and have not affected the experience of the company as between the two sections, I will not consider them in the figures here given. There are two important points to be noted:

1st. That there has been no marked difference in the number of policies in force in the two sections.

2d. The average amount of the policies in each section has been about the same, so that the comparison of the experience would seem to be as fair a one as could possibly be made.

With this explanation, I will now give the figures on male lives; non-abstainers; whole life policies; "transfers to" excluded:

Total number of years of exposure to risk, all ages	466,943.
Expected deaths by O^M table	8,911.
Actual deaths	8,947.
Per cent. of actual to expected	100.4

Male lives; abstainers; whole life policies; "transfers to" excluded:

Total number of years of exposure to risk	398,010.
Expected deaths by O^M table	6,899.
Actual deaths	5,124.
Per cent. of actual to expected	74.3

The difference between the percentages of actual to expected deaths as between abstainers and non-abstainers is, therefore, 26.1%, and the rate of death among non-abstainers was 35% higher than on the lives of abstainers.

The information given in Mr. Moore's paper produced a lengthy discussion among the members of the Institute, and a few of the speakers made some effort to break the force of the conclusions which would naturally be arrived at from the figures here given. This effort was natural, for the showing here made

seemed almost like a comdemnation of the habits of a majority of Englishmen, and, possibly, also a majority of English women. In my judgment, however, these efforts were not successful. One of the suggestions made was, that the lower mortality among abstainers might have arisen from their being a better class of lives in other respects than non-abstainers; but it was distinctly stated by Mr. Moore, in his paper, and by Mr. T. P. Whittaker, M. P.,—who, as the Chairman and Managing Director of the Company, was present as a visitor, and took part in the discussion—that there was practically no difference whatever between the two classes of lives in the General Section, and in the Temperance Section, except as regards their use of alcoholic beverages. Those in the Temperance Section were total abstainers, and those in the General Section, moderate drinkers, selected with the same care as is exercised in other life assurance companies. This fact is proved by the experience in the General Section, which shows approximately the same rate of mortality as the O^M table, which is based upon the experience of all British Companies for the thirty years between 1863 and 1893. Compared with the older experience, as shown by the H^M table, the death rate in the General Section shows a reduction of about 5½%, according to the figures given by Mr. Moore.

Instead of the lives in the Temperance Section being more carefully selected than those in the General Section, it was suggested in the discussion that the reverse was probably the case; for physical defects, or impairments, are always more likly to be overlooked by medical examiners in the case of total abstainers than with moderate drinkers. It seems to me, therefore, that Messrs. Manly and Hardy, and others who took part in the discussion, were correct in their statements that the figures derived from the long and extensive experience of the United Kingdom Temperance and General Provident Institution were conclusive in their demonstration that the effect of total abstinence from alcoholic beverages is beneficial to health, and conducive to longevity.

There are, however, many other testimonies to the effect that the use of alcoholic stimulants produces an increase in the rate of mortality. So destructive has been their effect upon the lives of the North American Indians, that under United States law, it is a penal offense to sell alcoholic beverages to an Indian. The

mortality among Africans and South Sea Islanders has also been so increased by the use of alcohol, that simply as a matter of humanity, the civilized nations of the world have united in their efforts to stop the sale of liquor in the Congo and in certain islands of the Pacific. While it is doubtless true that the deleterious effects of alcoholic beverages are more apparent among black men and red men than among white men, yet, as we have the assurance that all nations are made of the same blood, what is injurious to one, cannot be beneficial to another.

As further proof of the injurious effect of alcoholic beverages, as shown in the death rate, I would refer to the statistics which have been published from time to time, showing the percentages of mortality in the various occupations. These statistics have invariably shown a higher death rate among those engaged in the liquor business, from brewers down to bartenders, than among those engaged in other occupations, except such as are clearly defined as specially hazardous. The higher death rate among liquor dealers is so universally recognized by life assurance companies, that a number of them will not issue policies, even on the lives of the richest brewers, upon any terms, and not one of the companies, to my knowledge, admits liquor dealers upon as advantageous conditions as those engaged in other ordinary occupations.

As an example of the restrictions in this respect, I would quote the rules as given in a circular sent to the agency force of a prominent United States company. This circular reads almost like a temperance document, and yet it is simply sent out as a matter of business, because statistics show that owing to what might be called the very atmosphere by which liquor dealers are surrounded the mortality among them is higher than among those engaged in occupations which do not involve the handling of alcoholic beverages. This circular reads as follows:

"The number of applications received from persons engaged in the sale or manufacture of liquor has increased so rapidly that we find it necessary to call the attention to the rule on page 345 of the 'Blue Book' regarding this class of business, in order that unnecessary declinations may be avoided. This rule is as follows:

"Bartenders..... Not Taken.

"Saloon keepers, generally, not taken, but best of this class may be accepted on 10 or 15 Year Endowments only.

"Commercial Travelers (Salesmen).. $5 per thousand extra.
"Brewers (unless seldom at their breweries)..... $5 per thousand extra.
"Employees in breweries..... $5 per thousand extra.
"Wholesale dealers if apparently unaffected by their business..... Free
"Restaurant keepers and waiters selling liquor..... $5 per thousand extra.

"(In this whole class, the habits, past and present, and appearance will be carefully considered.)

"Please note that bartenders are *positively not taken*, and saloon keepers *tending bar occasionally* are therefore unacceptable. Only saloon keepers of the best class, very temperate in their habits, not tending bar, and enjoying the best moral surroundings which the business permits, will be taken at all, and only on 10 and 15 Year Endowment G. C. V. policies. It is useless to present cases not falling within these limits, as not only will the applications be declined, but the cost of medical examination may be charged against the agents for their failure to observe the rule.

"It is also noted that there is a growing tendency to present the applications of brewery employees for acceptance without extra premium. In order that there may be no misunderstanding on this subject, agents are hereby informed that the only persons employed directly in connection with breweries who will be accepted without extra premium, are the financial officers or officials seldom at the breweries, and employees such as bookkeepers, etc., whose work is performed in a building apart from the brewery proper, or the place where the product is stored or kept in bulk.

"General store-keepers everywhere, handling liquor at retail as part of their business, if taken at all, will hereafter be limited to dividend accumulation Endowment policies with not more than twenty years to run, but no policies with return of premium or with the Indemnity or Mortuary dividend feature will be issued to this class of applicant. This limitation is made necessary by the very excessive rate of mortality found to exist among persons so employed."

In connection with the rules and practice of companies, it may be well to state here, that an applicant for assurance is never refused because he is a total abstainer, while others are being

constantly rejected because of their excessive indulgence in alcoholic beverages, as shown in the results of medical examinations, or the reports made by inspectors of risks under consideration. A careful investigation is made of every applicant's drinking habits, because of the unquestionable fact that intemperance is one of the chief causes of a high rate of mortality, and a man known to drink to excess, can rarely obtain a policy in any company, even under the most disadvantageous conditions.

By many, however, who acknowledge and condemn the evils of intemperance, it has been generally supposed, that what is called a moderate use of alcoholic stimulants, is not only harmless, but even beneficial. It is largely for their benefit that this paper has been written, for while the primary service to be performed by Societies of Actuaries is somewhat technical in its nature, and, therefore, uninteresting to outsiders, yet it is a happy combination when statistics can not only furnish valuable information to life assurance companies and experts, but also at the same time, teach a lesson which may be of material value to the general public. I, therefore, appreciate the words of Mr. R. P. Hardy, when, in discussing the paper giving the experience of the London Company, he said he considered that Mr. Moore "was highly privileged that, through the instrumentality of the Institute, he was the means of communicating to the world an important series of authentic facts in aid of the solution of a social problem of indisputable gravity."

The question of indulgence in alcoholic beverages is one of world-wide interest, and an authority as eminent as that of the "Grand Old Man" of England, William E. Gladstone, has declared that intemperance has been productive of more evil than war, pestilence and famine combined. To lessen this evil, even in the slightest degree, is, in my judgment, worthy of the efforts of an actuary, and I frankly confess that this object has not been lost sight of in the preparation of this paper.

Drunkenness is universally condemned and deplored, but all drunkards began as moderate drinkers, and the smaller the number of moderate drinkers, the smaller will be the number of drunkards whose habits render it impossible for them to obtain that protection for their families in case of their death, which life assurance companies would, otherwise, be glad and willing to furnish. A total abstainer can never become a drunkard, and I

believe that it is universally admitted that a drunkard can never be reformed, except by becoming a total abstainer.

If, therefore, it can be proved that total abstainers show a death rate which is materially lower than that experienced among the most select body of moderate drinkers, in regard to which statistics are obtainable, I submit that total abstainers have the best of the argument as regards the use of alcoholic beverages; and considering the care with which the risks of a life assurance company are selected, intemperate applicants being rigidly excluded, I contend in conclusion, that the facts mentioned in this paper show conclusively that the effect of total abstinence is to lower the death rate, and increase the average duration of human life.

A Graduation of the Specialized Mortality Experience.

ARTHUR HUNTER.

At the last meeting of the Actuarial Society Mr. Weeks called attention to the rather surprising fact that the sum of the deaths in all the specialized classes investigated is almost identical with the expected deaths according to the standard assumed by the Committee as representing the mortality upon American insured lives. Taking the entire ninety-eight classes, we find that the expected deaths by the Committee's standard are 142,092, while the actual deaths are 141,977,—being 99.92 per cent. of the expected. If, then, the Committee were approximately right in its assumption of the rates of mortality prevailing among American insured lives at large, may we not assume that, since the aggregate mortality of the specialized classes corresponds with this assumption, the rates shown by this aggregate mortality for the successive attained ages and insurance years would fairly represent the general rates among American insured lives?

This assumption appeared to me to make it worth while to measure the aggregate mortality of the specialized experience, according to insurance years, by the American Table, which is so largely employed in our every day work. The following table gives the resulting comparison of the expected with the actual deaths:

NINETY-EIGHT CLASSES MEASURED BY THE AMERICAN q_x.

Insurance Year	Expected Deaths	Actual Deaths	Ratio of Actual to Expected Deaths
1	23,929	12,964	54.2%
2	16,982	11,984	70.6
3	14,594	11,112	76.1
4	12,517	9,984	79.8
5	11,178	9,454	84.6
6	10,027	8,672	86.5
7	9,064	7,740	85.4
8	8,267	7,260	87.8
9	7,500	6,639	88.5
10	6,846	6,359	92.9
11	5,766	5,473	94.9
12	5,181	4,985	96.2
13	4,707	4,439	94.3
14	4,377	4,182	95.5
15	3,915	3,848	98.3
5th to 9th	46,036	39,765	86.4
10th to 15th	30,792	29,286	95.1
16th to 20th	13,897	14,432	103.8
21st to 30th	12,763	12,450	97.5
1st to 30th	171,510	141,977	82.8

A study of the foregoing table indicates that each of the first four insurance years has its distinctive mortality, but that in the period from the fifth to the ninth insurance years, inclusive, the rate of mortality is nearly the same at attained ages, and that the same is true in the period from the tenth to the fourteenth insurance years, inclusive. From the fifteenth to the thirtieth insurance years the mortality is practically 100 per cent. of the American,—the first five years of this period showing, however, about five per cent. higher than the last ten.

The foregoing suggests in general that if the Specialized Experience were to be graduated, seven distinct tables would be

called for, which would have the following relationship to the rate of mortality by the American Table:

Group	Insurance Years	Ratio of Actual to Expected Deaths
(*a*)	1	54.2%
(*b*)	2	70.6
(*c*)	3	76.1
(*d*)	4	79.8
(*e*)	5th to 9th inc.	86.4
(*f*)	10th to 14th "	94.6
(*g*)	15th to 30th "	100.5

We have constructed the seven tables thus suggested, graduating them according to the Makeham method. In our first trial, four groupings of fourteen ages each were made for groups (*a*) to (*d*) inclusive, of fifteen ages each for groups (*e*) and (*f*) inclusive, and of sixteen ages each for group (*g*). The rates of mortality for a given attained age do not, however, in every case increase in successive insurance years. It seems advisable for our present purposes that the rate of mortality should increase both by insurance years and by attained ages, and as we could not readily obtain this result by the usual process of applying the Makeham graduation, the following method was adopted:

A skeleton table of the rate of mortality for quinquennial ages for the successive insurance years was prepared, which, in all cases, satisfied the requirement that the mortality should increase both by insurance years and by attained ages. The constants which had been obtained by the previous methods of grouping by ages were then modified in order to produce as nearly as possible the rates of mortality laid down in the skeleton table. The three constants, log. *c*, log. *g* and log. *s*, had only to be considered, as log. *k* is not required in obtaining the q_x. It was necessary to make a large number of trials in order to satisfy the requirements of the skeleton table, and also to produce rates of mortality which would result in the expected deaths being reasonably close to the actual deaths. I am greatly indebted to Mr. Frank B. Lucas for his work in connection with the graduation of these tables.

As the graduated rates of mortality may be used by some companies as their standard for measuring their expected deaths, I have given the constants used. Tables of the expected and actual deaths are also submitted, in order that the closeness of the graduation may be properly judged. In order to shorten the tabular work these tables are only presented for quinquennial age groups.

Insurance Years.

	1st	2nd	3rd	4th
Log. c —	0.04631153	0.04381072	0.04300000	0.04190000
Log. g —	$\bar{1}$.9999066	$\bar{1}$.9998387	$\bar{1}$.9998000	$\bar{1}$.9997599
Log. s —	$\bar{1}$.9984482	$\bar{1}$.9979838	$\bar{1}$.9978982	$\bar{1}$.9978861

	5th to 9th Inclusive	10th to 19th Inclusive	15th to 30th Inclusive
Log. c —	0.04141250	0.04001250	0.03907
Log. g —	$\bar{1}$.99970015	$\bar{1}$.99960000	$\bar{1}$.9995082
Log. s —	$\bar{1}$.9979100	$\bar{1}$.9979171	$\bar{1}$.9979632

There is a close resemblance between the constants of the British Offices Experience on Whole Life Participating Insurances on male lives, excluding the first five years of insurance $O^{M(5)}$ and the Classified Experience for the 15th to the 30th insurance years. The common logarithms of constants are as follows:

$O^{M(5)}$	Classified Experience for 15th to 30th years.
Log. c — 0.0390000	Log. c — 0.0390700
Log. g — $\bar{1}$.9994980	Log. g — $\bar{1}$.9995082
Log. s — $\bar{1}$.9974425	Log. s — $\bar{1}$.9979632

There was no intention of making the constants so nearly alike ; in fact, the similarity was not noticed until this paper was prepared.

TABLE I.

Rates of Mortality for 98 Classes—Makeham Graduation.

Attained Age	Policy Years.						
	1st	2d	3d	4th	5th to 9th	10th to 14th	15th to 30th
15	.00369						
16	.00370	.00483					
17	.00371	.00485	.00508				
18	.00373	.00487	.00511	.00519			
19	.00375	.00490	.00514	.00522	.00522		
20	.00377	.00493	.00517	.00526	.00526		
21	.00379	.00496	.00521	.00530	.00531		
22	.00382	.00499	.00525	.00534	.00536		
23	.00385	.00503	.00529	.00539	.00542		
24	.00388	.00507	.00534	.00544	.00548	.00559	
25	.00391	.00512	.00539	.00550	.00555	.00567	
26	.00395	.00517	.00545	.00557	.00562	.00575	
27	.00400	.00523	.00552	.00564	.00570	.00585	
28	.00404	.00529	.00559	.00572	.00579	.00595	
29	.00410	.00536	.00567	.00581	.00589	.00606	.00612
30	.00416	.00544	.00576	.00590	.00600	.00619	.00625
31	.00422	.00553	.00585	.00601	.00612	.00632	.00640
32	.00430	.00562	.00596	.00613	.00625	.00647	.00657
33	.00438	.00573	.00608	.00626	.00640	.00663	.00674
34	.00447	.00584	.00621	.00640	.00656	.00681	.00694
35	.00457	.00597	.00635	.00655	.00673	.00701	.00715
36	.00469	.00611	.00651	.00672	.00693	.00722	.00738
37	.00481	.00627	.00669	.00691	.00714	.00746	.00763
38	.00495	.00644	.00688	.00712	.00737	.00771	.00791
39	.00511	.00664	.00709	.00735	.00763	.00799	.00822
40	.00528	.00685	.00733	.00760	.00791	.00830	.00855
41	.00548	.00708	.00759	.00788	.00822	.00864	.00891
42	.00569	.00734	.00787	.00819	.00857	.00901	.00931
43	.00593	.00763	.00819	.00852	.00894	.00942	.00974
44	.00619	.00795	.00854	.00889	.00935	.00987	.01022
45	.00649	.00830	.00893	.00930	.00981	.01036	.01074
46	.00682	.00869	.00935	.00975	.01031	.01089	.01131
47	.00718	.00912	.00982	.01025	.01086	.01148	.01193
48	.00759	.00959	.01034	.01079	.01146	.01213	.01261
49	.00804	.01012	.01091	.01139	.01213	.01283	.01335
50	.00854	.01070	.01154	.01205	.01286	.01360	.01417
51	.00910	.01134	.01224	.01278	.01366	.01445	.01505
52	.00972	.01205	.01301	.01357	.01454	.01538	01.603
53	.01041	.01283	.01385	.01445	.01551	.01640	.01709
54	.01118	.01370	.01479	.01542	.01658	.01751	.01825
55	.01203	.01466	.01582	.01648	.01775	.01873	.01951
56	.01298	.01572	.01696	.01765	.01903	.02006	.02090
57	.01403	.01689	.01821	.01894	.02044	.02153	.02241
58	.01520	.01818	.01959	.02036	.02200	.02313	.02407

RATES OF MORTALITY FOR 98 CLASSES—MAKEHAM GRADUATION.
Continued.

ATTAINED AGE	POLICY YEARS.						
	1st	2d	3d	4th	5th to 9th	10th to 14th	15th to 20th
59	.01650	.01960	.02112	.02191	.02370	.02488	.02587
60	.01795	.02118	.02280	.02363	.02557	.02680	.02784
61	.01955	.02292	.02465	.02551	.02762	.02890	.02999
62	.02134	.02484	.02669	.02757	.02988	.03119	.03234
63	.02332	.02696	.02894	.02985	.03235	.03370	.03491
64	.02551	.02930	.03141	.03234	.03507	.03645	.03771
65	.02795	.03188	.03414	.03508	.03804	.03945	.04076
66	.03066	.03473	.03714	.03809	.04131	.04273	.04409
67	.03366	.03787	.04044	.04140	.04489	.04632	.04772
68	.03699	.04134	.04407	.04502	.04881	.05023	.05167
69	.04068	.04515	.04807	.04900	.05310	.05451	.05598
70	.04477	.04935	.05246	.05336	.05780	.05917	.06067
71		.05398	.05728	.05814	.06295	.06426	.06577
72			.06258	.06338	.06857	.06981	.07133
73				.06911	.07473	.07586	.07737
74					.08145	.08244	.08393
75					.08878	.08961	.09106
76					.09678	.09740	.09879
77					.10551	.10587	.10718
78					.11500	.11506	.11626
79						.12504	.12610
80						.13584	.13674
81						.14754	.14823
82						.16018	.16062
83						.17383	.17404
84							.18833
85							.20378
86							.22029
87							.23813
88							.25689
89							.27741
90							.29865
91							.32102
92							.34580
93							.37088
94							.39669
95							.42466
96							.45238
97							.49275
98							.54286
99							.62500
100							.83333
101							1.00000

TABLE II.

First Insurance Year.

Attained Ages	Expected Deaths	Actual Deaths	Error		Accumulated Error
15–19	86	108	— 22	—	22
20–24	763	759	+ 4	—	18
25–29	1,665	1,631	+ 34	+	16
30–34	2,132	2,039	+ 93	+	109
35–39	2,161	2,110	+ 51	+	160
40–44	1,937	1,993	— 56	+	104
45–49	1,580	1,571	+ 9	+	113
50–54	1,229	1,321	— 92	+	21
55–59	865	917	— 52	—	31
60–64	421	399	+ 22	—	9
65–70	128	116	+ 12	+	3
Total,	12,967	12,964	+ 3	+	3

Second Insurance Year.

16–20	69	91	— 22	—	22
21–25	607	697	— 90	—	112
26–30	1,456	1,380	+ 76	—	36
31–35	1,959	1,873	+ 86	+	50
36–40	2,026	1,863	+ 163	+	213
41–45	1,844	1,923	— 79	+	134
46–50	1,520	1,496	+ 24	+	158
51–55	1,170	1,308	— 138	+	20
56–60	821	843	— 22	—	2
61–65	398	415	— 17	—	19
66–71	116	95	+ 21	+	2
Total,	11,986	11,984	+ 2	+	2

Third Insurance Year.

17–21	56	64	— 8	—	8
22–26	517	569	— 52	—	60
27–31	1,281	1,219	+ 62	+	2
32–36	1,770	1,729	+ 41	+	43
37–41	1,873	1,907	— 34	+	9
42–46	1,735	1,578	+ 157	+	166
47–51	1,460	1,468	— 8	+	158
52–56	1,133	1,224	— 91	+	67
57–61	795	855	— 60	+	7
62–66	384	405	— 21	—	14
67–72	111	94	+ 17	+	3
Total,	11,115	11,112	+ 3	+	3

TABLE II—*Continued.*

FOURTH INSURANCE YEAR.

Attained Ages	Expected Deaths	Actual Deaths	Error	Accumulated Error
18–22	46	54	— 8	— 8
23–27	425	451	— 26	— 34
28–32	1,100	1,068	+ 32	— 2
33–37	1,564	1,491	+ 73	+ 71
38–42	1,676	1,616	+ 60	+ 131
43–47	1,583	1,669	— 86	+ 45
48–52	1,353	1,379	— 26	+ 19
53–57	1,048	1,063	— 15	+ 4
58–62	734	740	— 6	— 2
63–67	354	349	+ 5	+ 3
68–72	98	104	— 6	— 3
TOTAL,	9,981	9,984	— 3	— 3
FIFTH TO NINTH INSURANCE YEARS, INCLUSIVE.				
19–23	68	57	+ 11	+ 11
24–28	809	884	— 75	— 64
29–33	2,946	3,018	— 72	— 136
34–38	5,162	5,114	+ 48	— 88
39–43	6,284	6,120	+ 164	+ 76
44–48	6,548	6,414	+ 134	+ 210
49–53	6,026	6,089	— 63	+ 147
54–58	5,059	5,125	— 66	+ 81
59–63	3,801	3,849	— 48	+ 33
64–68	2,248	2,327	— 79	— 46
69–73	712	674	+ 38	— 8
74–78	107	94	+ 13	+ 5
TOTAL,	39,770	39,765	+ 5	+ 5
TENTH TO FOURTEENTH INSURANCE YEARS, INCLUSIVE.				
24–28	26	26	0	0
29–33	355	337	+ 18	+ 18
34–38	1,526	1,646	— 120	— 102
39–43	2,947	2,836	+ 111	+ 9
44–48	3,921	3,716	+ 205	+ 214
49–53	4,387	4,327	+ 60	+ 274
54–58	4,203	4,264	— 61	+ 213
59–63	3,649	3,784	— 135	+ 78
64–68	2,635	2,649	— 14	+ 64
69–73	1,373	1,429	— 56	+ 8
74–78	364	389	— 25	— 17
79–83	44	35	+ 9	— 8
TOTAL,	25,430	25,438	— 8	— 8

TABLE II—*Continued.*

FIFTEENTH TO THIRTIETH INSURANCE YEARS.

Attained Ages	Expected Deaths	Actual Deaths		Error		Accumulated Error
29–33	12	14	—	2	—	2
34–38	167	150	+	17	+	15
39–43	890	829	+	61	+	76
44–48	2,248	2,170	+	78	+	154
49–53	3,818	3,641	+	177	+	331
54–58	5,051	5,177	—	126	+	205
59–63	5,556	5,699	—	143	+	62
64–68	5,381	5,488	—	107	—	45
69–73	4,194	4,295	—	101	—	146
74–78	2,386	2,305	+	81	—	65
79–83	873	818	+	55	—	10
84–88	147	137	+	10		0
89–93	8	7	+	1	+	1
TOTAL,	30,731	30,730	+	1	+	1

The Makeham method necessarily yields a rising rate of mortality for each insurance year, from the youngest age up, but there is considerable doubt whether this corresponds with the facts in every body of insured lives. There is reason for thinking that the mortality among insured lives is heavier at the youngest ages of entry than at the next subsequent ages; in fact, Dr. Sprague proved that this condition exists in the H^M Experience. His Select Tables, which were graduated by the graphic method, give in the first calendar year a rate of mortality decreasing from age twenty to age twenty-seven, and it is not until the age of thirty-six is reached that the rate of mortality exceeds that at twenty; and all the calendar years in his Tables present a similar appearance.

It is an interesting question whether or not the same state of things holds good in the Specialized Investigation, and I have made a second graduation of the data according to a method likely to furnish the answer to this particular question. The method is the following:

Sum the unadjusted probability of dying for every five ages, then sum these totals in groups of five, repeating the process a third time, and dividing each of the values thus obtained by 125. (With slight modifications explained in my note contained in

Mr. Weeks' paper of last May, the Woolhouse graduation can be obtained directly from the foregoing process.) Table III gives the unadjusted probability of dying for attained ages fifteen to forty inclusive ; and Table IV gives the graduated probability calculated according to the above method for ages twenty-one to forty—the former being the youngest obtainable by that process. Inspection of Table IV will show that during the earlier years of insurance the youngest ages show a higher rate of mortality than the next subsequent ages, and the inference from this fact, as well as from the corresponding feature of the H^M Experience, is that it is harder to select first-class lives among men under twenty-five than among men after that age.

TABLE III—NINETY-EIGHT CLASSES—UNGRADUATED RATE OF MORTALITY.

Attained Age.	POLICY YEARS.										
	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th to 30th inclusive
15	.00287										
16	.00813	.00409									
17	.00660	.00330	.00161								
18	.00373	.00390	.00805	.00759							
19	.00444	.00874	.00630	.00388	.00682						
20	.00434	.00694	.00656	.00712	.00000	.00000					
21	.00357	.00569	.00540	.00751	.00106	.01676	.00294				
22	.00393	.00572	.00468	.00558	.00413	.00370	.00449	.00000			
23	.00396	.00498	.00597	.00538	.00615	.00375	.00148	.00813	.00400		
24	.00360	.00592	.00700	.00588	.00650	.01093	.00466	.00730	.00641	.00467	
25	.00368	.00630	.00671	.00654	.00608	.00661	.00545	.00760	.00647	.01145	.00578
26	.00420	.00513	.00489	.00526	.00718	.00726	.00516	.00958	.00357	.00765	.00581
27	.00392	.00556	.00591	.00611	.00654	.00558	.00645	.00472	.00973	.00447	.00331
28	.00422	.00471	.00570	.00536	.00465	.00637	.00693	.00581	.00308	.00286	.00790
29	.00361	.00470	.00551	.00534	.00609	.00587	.00572	.00620	.00646	.00460	.00552
30	.00392	.00516	.00480	.00646	.00539	.00699	.00544	.00525	.00447	.00565	.00644
31	.00468	.00584	.00535	.00558	.00654	.00569	.00780	.00604	.00684	.00613	.00572
32	.00401	.00571	.00599	.00594	.00694	.00735	.00539	.00600	.00729	.00690	.00689
33	.00392	.00544	.00531	.00547	.00745	.00667	.00583	.00455	.00646	.00623	.00577
34	.00408	.00477	.00629	.00714	.00686	.00620	.00699	.00581	.00544	.00630	.00977
35	.00473	.00569	.00630	.00620	.00675	.00708	.00635	.00655	.00789	.00925	.00729
36	.00478	.00525	.00648	.00634	.00661	.00690	.00761	.00664	.00750	.00550	.00758
37	.00460	.00575	.00738	.00620	.00677	.00680	.00608	.00719	.00801	.00841	.00719
38	.00450	.00613	.00676	.00696	.00721	.00656	.00710	.00792	.00789	.00836	.00820
39	.00492	.00636	.00665	.00659	.00722	.00825	.00758	.00723	.00776	.00658	.00799
40	.00540	.00625	.00756	.00764	.00795	.00836	.00688	.00730	.00674	.00611	.00812

TABLE IV.

Ninety-Eight Classes—Graduated Rates of Mortality.

Special Graduation—See explanation.

Attained Age.	POLICY YEARS.										
	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th to 30th inclusive.
21	.00417										
22	.00402	.00587									
23	.00391	.00579	.00593								
24	.00389	.00568	.00592	.00593							
25	.00390	.00555	.00588	.00587	.00570						
26	.00392	.00544	.00582	.00582	.00592	.00682					
27	.00395	.00535	.00572	.00578	.00605	.00665	.00562				
28	.00399	.00529	.00562	.00577	.00610	.00652	.00589	.00632			
29	.00403	.00528	.00557	.00579	.00614	.00647	.00609	.00609	.00593		
30	.00407	.00528	.00555	.00584	.00623	.00647	.00620	.00591	.00602	.00568	
31	.00412	.00531	.00560	.00590	.00639	.00650	.00626	.00581	.00619	.00590	.00649
32	.00418	.00535	.00572	.00600	.00654	.00656	.00634	.00581	.00639	.00621	.00678
33	.00426	.00541	.00589	.00610	.00669	.00664	.00643	.00592	.00664	.00657	.00705
34	.00435	.00547	.00610	.00622	.00680	.00674	.00652	.00612	.00687	.00685	.00729
35	.00446	.00556	.00632	.00634	.00690	.00684	.00664	.00636	.00710	.00709	.00748
36	.00461	.00571	.00654	.00649	.00699	.00699	.00678	.00666	.00731	.00728	.00766
37	.00477	.00591	.00674	.00666	.00711	.00720	.00692	.00694	.00750	.00743	.00778
38	.00496	.00614	.00693	.00688	.00730	.00744	.00709	.00723	.00764	.00755	.00792
39	.00518	.00644	.00710	.00717	.00754	.00771	.00728	.00752	.00780	.00776	.00807
40	.00541	.00676	.00724	.00750	.00783	.00799	.00753	.00783	.00796	.00800	.00827

It is by this time generally recognized that the true rate of mortality in the earliest insurance years is not represented by direct ratios of the American Table; that, for instance, the assumption that the mortality at all ages will be fifty per cent. of the American in the first year, sixty per cent. in the second year, etc., is materially wide of the facts, the true percentages being considerably higher at the older ages than at the younger. The Committee recognized this truth in its adjustment of the assumed standard mortality table, which was as follows:

For the purpose of summing the expected losses for all the years of duration, certain percentages of the loss expected by the standard table have been taken as expressing approximately the normal loss during the first five years of insurance, as follows:

Ages at Entry	Year One	Year Two	Year Three	Year Four	Year Five
15 to 28	45	64	79	90	97
29 to 42	50	68	82	92	98
43 to 56	55	72	85	94	99
57 to 70	60	76	88	96	100

We are now in a position to show corresponding percentages to the above, on the assumption of the American Table as a standard, and these percentages are as follows:

Ages at Entry	Year One	Year Two	Year Three	Year Four	5 to 9	10 to 12	15 to 30
15 to 28	48	66	68	71	74	82	89
29 to 42	51	65	71	75	82	91	105
43 to 56	61	80	85	89	95	101	103
57 to 70	71	79	90	88	96	100	93

A very interesting comparison can be made between the mortality of the new British Offices Experience and the Specialized Experience, as I have the expected deaths by the American q_x for both these Experiences. The expected deaths have been calculated for the aggregate experience of the British Offices

comprising Temporary Insurances, Whole Life, Limited Payment and Endowment Insurances on both Participating and Non-participating plans.

RATIO OF ACTUAL TO EXPECTED DEATHS BY THE AMERICAN RATE OF MORTALITY.

Insurance Years	Actuarial Society's Specialized Investigation	Aggregate Experience British Offices	Excess (or Deficit) of Specialized over Br. Offices
1	54%	43%	+ 11%
2	71	63	+ 8
3	76	71	+ 5
4	80	77	+ 3
5	85	82	+ 3
6	87	85	+ 2
7	85	91	— 6
8–10	90	99	— 9
11–20	99	105	— 6
21–30	98	103	— 5
All Years	83	89	— 6

In making any deductions from the foregoing we must always bear in mind that the mortality experience of the British Offices on Limited Payment Life and Endowment Insurance Policies was very low, and that accordingly the relative proportion of these plans of insurance in the aggregate experience has a great influence on the total mortality. We are probably justified, however, in drawing two general deductions from the foregoing: First, that the mortality in the early policy years in Britain is lower than in the United States; and, second, that the mortality in the later policy years (probably after the sixth) is higher in Britain than in this country.

On the Repayment of Loans by Contingent Instalments.

BY HUGH W. ROBERTSON.

For certain purposes it is sometimes required that a loan be extinguished by a series of uniform payments, to be made at equal intervals throughout a given term if the borrower survive, but to cease at his death should that event occur before the expiration of the term. In order that such an arrangement may be carried out it is usually necessary to effect an insurance on the life of the borrower which shall secure to the lender any principal and interest outstanding at the death of the former. The principal plans of insurance available for this purpose are the following:

(*a*) A single premium endowment insurance.
(*b*) An annual premium endowment insurance.
(*c*) A single premium decreasing term insurance.
(*d*) A yearly renewable decreasing term insurance.

It is shown in the present paper under which of the above plans the insurance necessary may be most economically secured, and suitable formulae are given for computing the amount of the periodical payment, and such other values as may be required.

In the discussion that follows it is assumed that the nominal amount borrowed is a unit, to be repaid by equal instalments falling due at the beginning of each year for a period of n years if the borrower survive, but to cease at his death should that event occur sooner. It is also assumed throughout that, in any expression, terms or factors which involve the rate of interest only, are based on the rate supposed to be earned by the lender.

It will in the first place be convenient to state and demonstrate the formulae referred to above. For this purpose let X represent throughout the amount per annum of the periodical payment.

CASE (*a*).

Let A^1 represent the office single premium for an *n*-year endowment insurance of $1 + A^1$ on the life of the borrower.

Then, in this case,

$$X = d(1 + A^1).$$

For the nominal amount advanced by the lender is in this case $1 + A^1$, but, as the whole of the first instalment is applicable to the repayment of principal, the actual amount advanced is only $1 + A^1 - X = (1 - d)(1 + A^1) = v(1 + A^1)$, which under this arrangement is to remain entirely outstanding for *n* years, or until the prior death of the borrower. As one year's interest on $v(1 + A^1)$ is $iv(1 + A^1) = d(1 + A^1)$, it follows that each instalment after the first will exactly meet the interest falling due at the time the instalment is payable, while the proceeds of the insurance policy will provide for the repayment at the end of *n* years, or at the prior death of the borrower, of the actual amount advanced, with one year's interest thereon, namely,

$$(1 + i)\, v\, (1 + A^1) = 1 + A^1.$$

CASE (*b*).

Let P^1 represent the office annual premium for an *n*-year endowment insurance of a unit on the life of the borrower.

Then, in this case,

$$X = P^1 + d.$$

For the nominal amount advanced by the lender is in this case $1 + P^1$, but as the whole of the first instalment is applicable to the repayment of principal, the actual amount advanced is only $1 + P^1 - X = 1 - d = v$, which under this arrangement is to remain entirely outstanding for *n* years, or until the prior death of the borrower. As one year's interest on *v* is $iv = d$, it follows that each instalment after the first will exactly meet the interest falling due at the time the instalment is payable, and also the renewal premium on a policy, the proceeds of which will provide for the repayment at the end of *n* years, or at the prior death of the borrower, of the actual amount advanced, with one year's interest thereon, namely, $(1 + i)\, v = 1$.

CASE (*c*).

Let A^1 represent the office single premium for an insurance on the life of the borrower of the principal and interest outstanding at the end of each year.

Then, in this case,

$$X = \frac{1 + A^1}{a_{\overline{n}|}}$$

For the actual amount advanced by the lender is in this case $1 + A^1 - X$, and, therefore, the principal and interest outstanding at the end of the first year, is

$$(1 + A^1 - X)(1 + i)$$
$$= (1 + A^1)(1 + i) - X(s_{\overline{1}|} - 1) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

assuming, as will be done in all similar cases, that the borrower is living.

Deducting from (1) the instalment payable at the beginning of the second year, and adding one year's interest to the remainder, we obtain

$$\{(1 + A^1)(1 + i) - X s_{\overline{1}|}\}(1 + i)$$
$$= (1 + A^1)(1 + i)^2 - X(s_{\overline{2}|} - 1) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

for the principal and interest outstanding at the end of the second year.

Proceeding in this way we find that the principal and interest outstanding at the end of the t^{th} year, is

$$(1 + A^1)(1 + i)^t - X(s_{\overline{t+1}|} - 1) \ldots\ldots\ldots\ldots (3)$$

Now as the entire loan is to be repaid in n years, we have

$$(1 + A^1)(1 + i)^n - X(s_{\overline{n+1}|} - 1) = 0$$

and, therefore,

$$X = \frac{(1 + A^1)(1 + i)^n}{s_{\overline{n+1}|} - 1}$$
$$= \frac{1 + A^1}{a_{\overline{n}|}}$$

Substituting this value for X in (3), we obtain $\frac{1 + A^1}{a_{\overline{n}|}} \cdot a_{\overline{n-t}|}$ as a final expression for the principal and interest outstanding at the end of the t^{th} year.

From this it follows that

$$A^1 = \frac{1 + A^1}{a_{\overline{n}|}} \cdot \Sigma^n_{1\,t-1|1} A^1 \cdot a_{\overline{n-t}|}$$
$$= \frac{\Sigma^n_{1\,t-1|1} A^1 \cdot a_{\overline{n-t}|}}{a_{\overline{n}|} - \Sigma^n_{1\,t-1|1} A^1 \cdot a_{\overline{n-t}|}}$$

where ${}_{t-1|1}A^1$ represents the loaded single premium for a $(t-1)$-year deferred, 1-year term insurance of a unit.

CASE (d).

Let P_t^1 represent the office yearly renewable term premium for an insurance on the life of the borrower of the principal and interest outstanding at the end of the t^{th} year.

Let $\pi^1{}_t$ represent the corresponding premium per unit.

Then, in this case,

$$X = \frac{1}{1+(v-\pi_1^1)+(v-\pi_1^1)(v-\pi_2^1)+\ldots+(v-\pi_1^1)(v-\pi_2^1)\ldots(v-\pi^1{}_{n-1})}$$

For the actual amount advanced by the lender is in this case $1 + P_1^1 - X$, and, therefore, the principal and interest outstanding at the end of the first year, is

$$(1 + P_1^1 - X)(1 + i) \ldots\ldots\ldots (1)$$

From this it follows that

$$P_1^1 = \pi_1^1 (1 + P_1^1 - X)(1 + i)$$
$$= \frac{\pi_1^1 (1 - X)}{v - \pi_1^1}$$

and substituting this value for P_1^1 in (1), we obtain

$$\frac{1}{v-\pi_1^1} - X\frac{1}{v-\pi_1^1} \ldots\ldots\ldots (2)$$

as a final expression for the principal and interest outstanding at the end of the first year.

Deducting $(X - P_2^1)$ from (2), and adding one year's interest to the remainder, we obtain

$$\left\{\frac{1}{v-\pi_1^1} - X\left(\frac{1}{v-\pi_1^1}+1\right) + P_2^1\right\}(1+i) \ldots (3)$$

for the principal and interest outstanding at the end of the second year.

From this it follows that

$$P_2^1 = \pi_2^1 \left\{\frac{1}{v-\pi_1^1} - X\left(\frac{1}{v-\pi_1^1}+1\right) + P_2^1\right\}(1+i)$$
$$= \frac{\pi_2^1 \left\{\frac{1}{v-\pi_1^1} - X\left(\frac{1}{v-\pi_1^1}+1\right)\right\}}{v-\pi_2^1}$$

and substituting this value for P_2^1 in (3), we obtain

$$\frac{1}{v-\pi_1^1}\cdot\frac{1}{v-\pi_2^1} - X\left(\frac{1}{v-\pi_1^1}\cdot\frac{1}{v-\pi_2^1} + \frac{1}{v-\pi_2^1}\right)$$

as a final expression for the principal and interest outstanding at the end of the second year.

Proceeding in this way, we find that the principal and interest outstanding at the end of the t^{th} year, is

$$\frac{1}{v-\pi_1^{\,1}}\cdot\frac{1}{v-\pi_2^{\,1}}\cdots\cdots\frac{1}{v-\pi_t^{\,1}}-X\left(\frac{1}{v-\pi_1^{\,1}}\cdot\frac{1}{v-\pi_2^{\,1}}\cdots\cdots\right.$$

$$\left.\frac{1}{v-\pi_t^{\,1}}+\frac{1}{v-\pi_2^{\,1}}\cdot\frac{1}{v-\pi_3^{\,1}}\cdots\cdots\frac{1}{v-\pi_t^{\,1}}+\cdots\cdots+\frac{1}{v-\pi_1^{\,1}}\right)$$

Now, as the entire loan is to be repaid in n years,

$$\frac{1}{v-\pi_1^{\,1}}\cdot\frac{1}{v-\pi_2^{\,1}}\cdots\cdots\frac{1}{v-\pi_n^{\,1}}-X\left(\frac{1}{v-\pi_1^{\,1}}\cdot\frac{1}{v-\pi_2^{\,1}}\cdots\cdots\right.$$

$$\left.\frac{1}{v-\pi_n^{\,1}}+\frac{1}{v-\pi_2^{\,1}}\cdot\frac{1}{v-\pi_3^{\,1}}\cdots\frac{1}{v-\pi_n^{\,1}}+\cdots+\frac{1}{v-\pi_n^{\,1}}\right)=0$$

from which the value of X already given can be readily derived.

It is easy to show that, while the actual amount advanced by the lender is different in each of the above cases, the value of X is the same when the premiums are taken as net according to a given mortality table, and a rate of interest equal to the rate supposed to be earned by the lender.

Thus, we have,

$$(a)\ldots\ldots d\,(1+A^1)=d\left(1+\frac{A_{x\overline{n}|}}{1-A_{x\overline{n}|}}\right)=\frac{d}{1-A_{x\overline{n}|}}=\frac{1}{a_{x\overline{n}|}}$$

$$(b)\ldots\ldots P^1+d=P_{x\overline{n}|}+d=\frac{1}{a_{x\overline{n}|}}$$

$$(c)\ldots\frac{1+A^1}{a_{\overline{n}|}}=\frac{1+\dfrac{\Sigma_1^{\,n}{}_{t-1|1}A_x\cdot a_{\overline{n-t}|}}{a_{\overline{n}|}-\Sigma_1^{\,n}{}_{t-1|1}A_x\cdot a_{\overline{n-t}|}}}{a_{\overline{n}|}}=$$

$$\frac{1}{a_{\overline{n}|}-\Sigma_1^{\,n}{}_{t-1|1}A_x\cdot a_{\overline{n-t}|}}=\frac{1}{a_{x\overline{n}|}}$$

$$(d)\ldots\frac{1}{1+(v-\pi_1^{\,1})+(v-\pi_1^{\,1})(v-\pi_2^{\,1})+\cdots\cdots+(v-\pi_1^{\,1})(v-\pi_2^{\,1})\cdots\cdots(v-\pi^1{}_{n-1})}$$

$$=\frac{1}{1+v(1-q_x)+v^2(1-q_x)(1-q_{x+1})+\ldots+v^{n-1}(1-q_x)(1-q_{x+1})\ldots(1-q_{x+n-2})}$$

$$=\frac{1}{1+v_1\,p_x+v_2^{\,2}\,p_x+\cdots\cdots\cdots\cdots,\cdots\cdots+v_{n-1}^{n-t}\,p_x}$$

$$=\frac{1}{a_{x\overline{n}|}}$$

It will next be shown under which of the above plans X has its least value, when actual office premium rates enter into the calculations. In order to do this it will be sufficient to compare the values of X under the different plans for ages 25 and 35, with periods of 20 and 10 years respectively, and with an assumed interest rate of 6% per annum.

(*a*)

x	n	Office Single Premium, per unit, for an n-year Endowment Insurance (Non-participating).	X
25	20	0.59166	0.13861
35	10	.79415	.27496

(*b*)

x	n	Office Annual Premium, per unit, for an n-year Endowment Insurance (Non-participating).	X
25	20	0.04266	0.09926
35	10	.09485	.15145

(*c*)

x	n	Office Single Premium, per unit of loan, to insure the Outstanding Principal and Interest.	X
25	20	0.10253	0.09068
35	10	.06129	.14720

(*d*)

x	n	Office Yearly Renewable Term Premium, per unit.	X
25	20	0.01210	0.09097
26		.01220	
27		.01230	
28		.01240	
29		.01252	
30		.01264	
31		.01277	
32		.01291	
33		.01308	
34		.01325	
35	10	.01342	0.13601
36		.01363	
37		.01385	
38		.01411	
39		.01438	
40		.01469	
41		.01501	
42		.01538	
43		.01578	

The above results would indicate that, on the basis of loading here adopted, the insurance could be secured to best advantage under the Yearly Renewable Term plan. Besides the fact that under this plan the values of X are generally less than in the other cases, there are the additional advantages of the smaller initial outlay by the lender, and the smaller amount which would be required to pay off the loan in full at any time before the expiration of the period.

The calculation on the Yearly Renewable Term plan of extensive sets of the values of X, and of the corresponding amounts of Outstanding Principal and Interest, may be greatly facilitated by first forming D and N columns, where $\frac{D_{x+1}}{D_x} = v - \pi_x^1$. The values of X would then be readily obtainable by means of the formula $\frac{D_x}{N_x - N_{x+n}}$, and the amounts of Outstanding Principal and Interest at end of the t^{th} year by

$$\frac{D_x}{N_x - N_{x+n}} \times \frac{N_{x+t} - N_{x+n}}{D_{x+t}}$$

Abstract of the Discussion of Papers Read at the Previous Meeting.

ON THE PRINCIPLES WHICH SHOULD DETERMINE THE MAXIMUM SINGLE RISK, AND THE ACCEPTANCE OF EXCEPTIONAL CLASSES OF RISKS—W. S. NICHOLS. Vol. VIII., Page 1.

Written Discussion.

Mr. Crawford: Mr. Walter S. Nichols, in his review of the chapter of Mr. T. E. Young's book on the maximum single risk, etc., vigorously and almost convincingly brings forward views opposed to those hitherto commonly held about the subject. Have we thus far been in error, and have our theories of this question been "tainted with certain fallacies?"

The view presented by Mr. Young is the familiar one. I need not repeat it. Mr. Nichols, if I understand him, takes the position that a company may properly take a relatively large risk provided the company is sufficiently strong, has a sufficient margin of assets over liabilities to enable it to pay without inconvenience or embarrassment should the risk become a claim; and that a company so situated financially need not hesitate to take up such risk, even though it may not promptly secure a sufficiently large number of equally large risks so that they may of themselves mutually insure one another. This, I think, fairly epitomizes Mr. Nichols' opinion as appearing in the paper before us.

The illustration offered to show the assumed fallacy of Mr. Young's theory is not to my mind conclusive, and unless I miss the point is itself fallacious. The inferential hypothesis is that each of the constituent companies before the merger had a sufficient number of $5,000 risks to "average among themselves," to be, if I understand the phrase, mutual insurers of one another, but that each has not, measured by Mr. Nichols' test, sufficient surplus to enable it to take up a single $25,000 risk "without opening the door to serious fluctuation." Now, it is hardly logical to conclude that because the merger is five times as strong as each of the five constituent companies it may therefore assume the single risk resulting from the amalgamation, a risk five-fold the maximum carried, on a number sufficiently large to form an average, by each of the component parts; unless, as a result of the merging and the consequent increase in strength, greater reliability and increased public confidence, it may readily secure a class of such maximum risks sufficiently large to, among themselves, afford mutual protection.

The theoretical and fundamental principle in life insurance is this: Each insured member of a company contributes to the yearly mortality costs proportionately to the risk carried for him, and since the risk varies with the age of the insured, the kind and amount of policy (physical condition, moral hazard, etc., are foreign to this discussion), it follows that in the last analysis there should be a sufficient number of insurances of the same kind and amount at each age, within the desirable range of ages, so that each class so constituted may be an elemental insurer mutually of its members. Under the level premium plan, all members of each class as described pay in any year equal mortality costs, and their aggregate equals that of the incidental claims less their reserves.

A very elementary concrete example will illustrate: According to a certain standard mortality table, of 10,764 lives at age thirty-five, one hundred will fail before attaining age thirty-six, a ratio of 1 to 107.64. On the basis of $1,000 for each policy, ordinary life plan, and age thirty-five, the cost of insurance for the first year by the same table is $9.1837, whereas the contribution to meet payment of the one claim is $988.52, which, with the terminal reserve, $11.48 for the policy, is the $1,000 required to pay the claim. The illustration may be carried forward from year to year with like results.

It follows that a company composed of elemental classes as described, is on the highway to safety and success, it being assumed that the several constituent classes have each sufficient numbers so that lives failing therefrom may closely follow the mortality table used by the company to measure its risks.

But assume that one of the above class of 107.64 has a $5,000 policy, and that during the year it is his life that fails. Then we have:

106.64 at $9.1837 each contribute	$979.35
1 contributes	45.92
Reserve for the $5,000 policy	57.40
Total	$1,082.67

The total is insufficient to meet the assumed $5,000 claim, and the remainder, $3,917, becomes a draft upon surplus, Now it is not clear wherein this differs from the amalgamation assumed by Mr. Nichols, except in magnitude; but this, however, increases insurance costs, reserves on claims and number of claims proportionately without disturbing the values relatively.

Mr. Nichols advocates taking up a single abnormally large risk, or a few such risks, provided the company's surplus is large enough to stand the draft upon it should one or more of such risks be presented for payment. He by this virtually admits that the normal mortality costs plus the reserves on claims might not be adequate to discharge the claims. But that this balance should subsist is in accordance with insurance fundamentals.

Moreover, to rely upon surplus to help out in paying relatively large claims is subversive of its function, and an infringement upon equities, except possibly in strictly proprietary companies. Surplus, and I refer to so much of the excess of assets over liabilities as may properly be set aside as a margin of safety, is a provision against adverse contingencies, *unforeseen* and not otherwise provided for, happening adversely varient from *normal* assumptions, and not a provision to meet unusual drafts resulting from *abnormal* conditions *purposely* set up. To rely upon surplus for this purpose necessitates a larger safety fund than otherwise required, and a resulting sequestration of properly divisible surplus and hence a violation of equities.

The multi-millionaire at Monte Carlo may play the maximum hazard, but the man of moderate means on the board of trade should not attempt a corner in wheat. It does not, however, follow that because a company has large resources and large surplus its officers may properly take up a jumbo risk and by so doing put in jeopardy any portion of the trust funds committed to their care.

It is not to be expected that in practice a company may actually realize the theoretical ideals. There will be departures from the fundamental assumptions in the best regulated company, no matter how large and strong. The less the range of fluctuations, however, the more desirable it is. It would appear, then, inconsistent to introduce causes, the effect of which would be to enlarge the range of fluctuations, and certainly for a company to make its maximum so large as to render it probable that it would be unable to secure a sufficiently large class of such risks as to be mutally protective would be to introduce such a disturbing cause.

MR. OGDEN: The very interesting paper which Mr. Walter S. Nichols presented to this Society at its last meeting, in which he so forcibly expressed his views in relation to the method of determining the limitation of risk which a company would be justified in assuming, as being diametrically opposed to the conclusions reached by Mr. T. E. Young, former President of the British Institute of Actuaries, will, I am sure, furnish food for thought in the minds of many of us.

As Mr. Nichols has well said, "a small volume might be written on the mathematical principles involved in this complicated subject." But inasmuch as it is Mr. Nichols' paper which is under discussion, and not Mr. Young's, and on page 4 of said paper he tells us that "the actual amount of fluctuation which a company may treat as normal and which determines the maximum line which it may write is not a mathematical question at all," it will, I presume, be eminently proper to deal with the question from a purely practical standpoint, and treat the maximum limit as an arbitrary quantity which should be determined by business considerations alone. Mr. Nichols asserts that "the temporary excess of death losses or diminution of profits which a company can meet without injury to its standing or business is the proper limit to its normal fluctuations, and this is regulated by

the business sentiments or prejudices of its members and of the public." He then refers to other factors which should be considered in determining the risk of fluctuation, such as the actual percentage of expected loss and the nature of the causes themselves which produce that loss. It does not seem to me at all probable that Mr. Nichols' criticism of Mr. Young's position will be disputed, neither does it seem at all difficult to reconcile the position taken by Mr. Young with what would appear to be almost the opposite view of that taken by Mr. Nichols. A careful reading of the chapter referred to in Mr. Young's book on Insurance will disclose the fact that the subject of "Risk and its Limitation" is dealt with altogether from a theoretical standpoint, whereas the present paper under discussion discards the theoretical or mathematical side of the question almost entirely, and deals with the subject from a practical or business standpoint,

Had Mr. Young entered more fully into the discussion of the subject in hand, he would doubtless have touched on many of the practical considerations suggested by Mr. Nichols, and in the end would probably have arrived at the same conclusions or nearly the same conclusions as those arrived at by Mr. Nichols. In Mr. Young's statement "that the true basis for assessment of the limit of risk does not consist of the extent of the premium revenue or the magnitude of the assets which the company may possess, but solely relates to the number of policies of maximum amount which the nature and range of its business entitle it to expect," he might have amplified the same by suggesting the proviso, that if the surplus or profits of a company were considered large enough to warrant drawing upon this fund to meet that portion of the unexpected increased fluctuations of a few large risks which had not been provided for in the premiums received from the same class of risks, then and in that case only would a company be justified in accepting one or a few risks many times larger in amount than the average risk, but not of sufficient number to establish an average for themselves. But Mr. Nichols goes even further than this and asserts "that instead of seeking for enough of a certain class to average among themselves, it may be necessary to minimize the possible loss by limiting the number and amount of any specific class which shall be taken, and to seek an average by offsetting the uncertainty of one class against those of another." In this opinion Mr. Nichols practically concurs in the views by other eminent writers on this subject, and I might add, with the view held by all or nearly all of the Actuaries of the present day.

In a very able discussion on this subject in a paper read by Mr. Thomas Bond Sprague, before the British Institute of Actuaries on January 9th, 1866, he says, "can we then conclude generally that a company is justified in running the risk of having to pay a very large sum on the happening of any extremely improbable contingency?" . . . . . Continuing, he says, "It will no doubt be answered by some persons that a company would not be justified in undertaking insurances of the nature mentioned in the last paragraph, because

such insurances, if ever made, would be rare, and could never be sufficiently numerous to enable the company to obtain an average result from them." To this argument Mr. Sprague says, "I cannot myself allow any weight," and adds, "I can find no sufficient reason for thinking that it is necessary for an insurance company to have a sufficient number of each class of risks to form a separate average."

Mr. Sprague refers to a very interesting paper written and published in 1856 by the late Dr. Thomas Young, on the practical application of the Doctrine of Chances, as it regards the Subdivision of Risks. He entirely agrees with the final conclusions arrived at by Dr. Young, who sums the subject up in these words: "There is a common prejudice that it is disadvantageous for an office to take a single risk of any particular description; and it is sometimes said that if the adventure should happen to be unsuccessful, there would be no possible compensation from others of the same kind. There is, however, just the same chance that it would be compensated by others of a different kind."

It is evident, therefore, that as early as 1856 there were Actuaries who agreed with Mr. Nichols, and I think it safe to say that there are many who agree with him to-day—I know of one—and after reading the supplemental paragraph in the paper under discussion, we can be satisfied that Mr. Young himself can perceive no difficulty in accepting the views expressed by Mr. Nichols and the other gentlemen I have mentioned.

Oral Discussion.

Mr. Craig: Mr. Nichols in this paper has brought to his aid the laws of nature in the storms which we occasionally see the effects of, as well as feel, and of the deviation of a ray of light, to demonstrate that the law of fluctuation is general; and quotes somewhat copiously from the paper of Mr. Young, of which paper his is a criticism.

Mr. Young says: "That in a company having decided upon a maximum line of three thousand dollars, and having all its policies within that limit, in order to increase that maximum line to ten thousand dollars or any other figure, must have a reasonable expectation of securing enough of such ten thousand dollar policies to furnish an independent average,"

Then Mr. Nichols says: "If he is correct, a company, no matter how many policies for ten thousand dollars or under it may have on its books, or how large its assets, would not be justified in writing a single line for fifty thousand dollars, or any other excessive amount, unless it had a reasonable prospect of securing enough of such policies to maintain an independent average." He says later on: "The limit of the maximum line, instead of being dependent on obtaining sufficient policies of any particular class and being independent of the resources and general business of the company, depends almost solely on the latter, and the company may be justified in accepting a single large policy, though unable to secure another. As has been well

remarked to the writer by a member of this Society, the best test of a proper limit is the strain liable to be put upon the surplus by fluctuations. The principle involved is but another phase of the familiar doctrine of moral hazard. The capitalist of large resources can profitably engage in speculations which would be dangerous to the man of moderate means. He may profitably make a single venture which would be folly on the part of one having but limited funds."

The case, from Mr. Nichols' standpoint, seems thus to be pretty well put, and so clear that we can understand what he means. But, in connection with the latter part of his remark, the thought would naturally occur to one whether the executive officers of a company would be justified in putting themselves on a par wth a speculator of large means in dealing with a trust fund, and whether they could reconcile an inroad into the surplus of the company in order to minimize an excessive loss with a fair and rational administration of the general funds of the company?

As I look upon it, the surplus, particularly in a mutual company, is just as much a part of the money of the policy-holders as is the legal reserve, and as such should be administered with the same jealous care for their interests as any other part of the general fund.

There is a kind of rule which Mr. Nichols has laid down which I am not quite sure that I understand:

"The maximum line which any corporation can profitably write is largely determined by the least number of policies into which its aggregate risks could be divided and still furnish a satisfactory average."

Now, I do not know whether he means that the satisfactory average applies to the minimum number that the law of average could operate upon, or whether he means a satisfactory average amount. If he means a satisfactory average amount then it becomes an arbitrary rule which each company would have to decide for itself. It becomes simply a question of judgment as to what would be a maximum average amount which would be the largest that the company would be willing to carry.

If it is the number, the minimum number, divided into its aggregate risks that will furnish a satisfactory average amount, and if we assume that one thousand would be the minimum number that the law of average would satisfactorily operate upon, then the company having aggregate risks upon its books amounting to one thousand million dollars could profitably accept a single risk of one million dollars, and a company having risks upon its books amounting to one hundred million dollars could profitably accept a single risk of one hundred thousand dollars.

I don't know of any company having an aggregate amount of one thousand million dollars at risk that is willing to accept and carry without any reinsurance, a one million dollar risk on a single life. And if the largest companies not only having an aggregate amount of insurance in excess of a thousand million dollars, but also with ample

assets and surplus, do not feel justified in writing on a single risk, without reinsurance, up to the limit of one million dollars, then it is fair to assume that the combined judgment of the experienced men at the head of those institutions is against that rule.

There is one statement in Mr. Nichols' paper which is very true, "The danger of serious fluctuations from an average largely depends on the frequency with which a company must balance its accounts."

A company whose average writing is, say, fifteen hundred dollars, and one that has upon its books a risk of one hundred thousand dollars; if that one hundred thousand dollars becomes a claim, it does pinch. It pinches in the year in which that claim falls due. The company is not permitted to distribute that claim over a series of years. It must go in the year in which it falls due. Of course, if you make up your mortality covering a period of five years, then that amount does not appear so large, because it is spread over the five years. But, nevertheless, in treating with a single case, it is felt in the year in which it occurs, and it seemed to me that in the general discussion of this question, Mr. Nichols in applying the mathematical rules, applies them in a technical rather than in a practical sense.

In the everyday course of business, a policy of undue magnitude cannot be considered as having the cost of risk considered as the mathematical cost, because the man is an individual in that case standing alone, and not one of a sufficient number to constitute a fair average upon which the law of average can operate. That single individual in any one year will either live or he will die, and if he die, then the whole amount of the claim must be paid, and you cannot simply consider it at its pro rata cost.

There is one other point in Mr. Nichols' paper where he alludes to the fluctuations, the deviations from the normal that are liable to occur in nature, as from contagious diseases. I take it that if the ravages of a contagious disease were spread over a large territory, and so caused an abnormal strain on the death cost for a year, there would be no adverse criticism, because that would be something beyond the power of man to anticipate. It would be something that the executive officers could not guard against; it would be universally recognized that a great scourge had caused that result and nobody would complain, nobody would criticise, and even if the strain were so great as to cause insolvency, no unfavorable comment would be provoked.

But the case is different if an undue strain on the funds of the company has been caused by insufficient care in the selection of risks, as was the case in one company, the report of whose examination I read some time ago. In this company, the secretary, who seemed to be the whole thing, often overrode the medical director, when, in his superior judgment, he thought that a risk was a fit subject for insurance, and passed it, after it had been turned down by the medical director. And there were so many of those cases, it being the rule rather than the exception, that the inevitable result was that the rate of mortality continued to increase until the company became insolvent.

In this case insolvency was directly attributable to an undue strain on the funds of the company by reason of excessive mortality. The excessive mortality was due to the arbitrary rulings of one of the officers of the company. An excessive mortality, from a monetary point of view, may be caused by one large claim occurring each year, for a few years, as well as from an excessive death rate in the number of claims for average amounts. It is true that the probability of such a thing happening is very remote, but improbable things do sometimes happen, and when they do occur, the mathematical improbability of their occurrence will not restore matters to a normal condition.

It seems to me that it is not good business judgment to put a risk upon the books of the company in amount so far in excess of the average amount as to render the company liable to an excessive strain upon its funds, and I am constrained to say that Mr. Nichols has failed to convince me that his argument is sound, from a practical point of view. I would prefer to take a large enough number of any class to constitute an average, rather than to run the risk of having a few exceptionally large cases, the loss among which might prove a heavy strain upon the funds of the company.

MR. SHEPPARD: Mr. President, there is one practical question which I have not heard touched upon, either by Mr. Nichols or by the gentlemen who have criticised this paper, that is, just what office rules should be made by the company in case they decided to raise the limit on risks. Supposing it had fifteen thousand dollars as the limit, and it wished to raise the limit to twenty-five thousand dollars, on a single life. If they were to consider the question of applications on high premium policies, for instance, twenty-payment life, fifteen or twenty-year endowment, and only take those lives up to the full limit, this would have the three-fold advantage, first, of a guarantee of the good faith of the applicant that it was not merely to be taken out for speculation; secondly, the higher premium policy leading to higher reserves, in case of death the strain on the surplus would be less; and, thirdly, the high premium policy would cause the surplus to be more readily increased through excess of interest above valuation rate.

Taking those three things into consideration, I think that the company could very well increase its limit on the higher premium policies until it got such a number that they would be able to form a separate class and be treated by themselves, and then the lower premium policies could gradually be admitted.

The only other remark I wanted to make was in regard to what Mr. Craig said about the surplus being guarded, well cared for. I quite agree with that, but I think that we will all agree that one of the objects of the surplus is to act as a balance between excessive or under average mortality, so that a great loss in one year may be balanced by profits in the succeeding years, and that is especially so in this country where we have to take our assets and liabilities every year instead of every five years, as they do on the other side.

MR. HENDERSON: Mr. President, I believe it is the custom in certain military councils for the junior persons to speak first. Therefore I appear early in this discussion.

I simply wanted to voice an idea which occurred to me in the course of Mr. Ogden's remarks, and that is, it has appeared to me that if there is any essential difference, mathematical or practical, between Mr. Young and Mr. Nichols, it is not that Mr. Young was mathematical and Mr. Nichols practical, but rather that Mr. Young, probably in his haste, did not go as deeply into the mathematics of the problem as he might have gone if he had taken a little more time to the discussion.

Since receiving Mr. Nichols' paper, I have jotted down a few little suggestions, and it seems to me that this problem is entirely one of the fluctuation of the actual claims from the expected, and leaving out of account the different ages and assuming for the present that all are of one age, it seems to me that the condition of affairs could on that assumption be reduced to two factors, one which might be called the equivalent number of risks and the other the equivalent average amount. These two being so determined that the equivalent number multiplied into the equivalent average amount will equal the total amount at risk; and that the equivalent number multiplied into the square of the equivalent average amount will equal the sum of the squares of the amounts of actual risks.

Having then reduced the present status of the company to that position, the mathematical investigation is simplified, and I will simply advert to one or two points that seem involved in the result; and the first is, that the taking of large amounts will not in any way reduce the equivalent number of risks upon which the relative fluctuations depend unless this large amount is twice the equivalent average of the company's business already on its books. And the next point that comes up is, that granted that the maximum limit under discussion is larger than twice the equivalent average, by taking an increasing number of those large risks, the equivalent number will reduce to a minimum and then increase again, and that minimum is reached when the equivalent average amount has been raised to one-half of the maximum limit.

And the third point is, that if the average amount is A, say, and its present equivalent number is n, and that the executive officers considered that they could afford to reduce that equivalent number to l, the limit of insurance which could be fixed in order that the minimum point would not bring them below that value would be $2\ A/1 - \sqrt{1 - l/n}$.

In other words, if the company's present equivalent average is three thousand, and it considers that it could reduce its equivalent number by one per cent., that is, to 99/100 of its actual situation, then the limit which it would fix would be 20/9 of its present equivalent average. That would be 6,700 or thereabouts.

MR. MACAULAY: Mr. President, I heartily agree with the general principle laid down by Mr. Nichols, that the limit of risk which a company can safely take depends principally upon the amount of that company's surplus, and the annual probable increases to that surplus, and the frequency of dividing profits to its policy-holders. In other words, primarily and principally this question is a business one rather than a mathematical one.

There are one or two minor points which Mr. Nichols raises that I am not sure that I agree with, as for example, that remark to which Mr. Craig has called attention, that the maximum line which any corporation can profitably write is largely determined by the least number of policies into which its aggregate risks can be divided and still furnish a satisfactory average. I am not sure that I entirely understand that clause. With the main lines of Mr. Nichols' paper, I am in very hearty accord.

It seems to me that the practical and proper way of looking at this question is about as follows:

First of all, we should decide what amount of fluctuation in the amount of a company's surplus can be permitted without injuring the company's standing, or interfering with the profits which it would give to its policy-holders.

The next point to settle is to what extent the company is liable to fluctuation in its mortality in connection with the business which it at present has upon its books. Every company's death losses are subject to fluctuations more or less, even without increasing its maximum risk, and before we attempt to decide whether we can increase our maximum risk we first should ascertain what is the maximum fluctuation to which the company is already liable.

The next point is the extent to which that probable fluctuation, due to existing business, might be increased if we were to extend the limit of risk upon individual lives.

When we know to what extent the present probable fluctuations would be increased by the issue of a few policies of larger amounts we can then ascertain whether that total possible or probable fluctuation would exceed the amount of permissible fluctuation in the company's surplus, which we have already settled, and decide accordingly as to the amount of maximum individual risk.

Account would have to be taken very clearly of the number of large policies which would be likely to be issued.

I hardly think that Mr. Young's position can be sustained, that we would have to obtain a large enough number of policies to form an average of these large amounts. That is practically impossible in those large amounts. On the contrary, I think we will all agree, that if a company has only one risk of a large amount it is a great deal safer than if it has two. If the one solitary life carrying the large amount of assurance were to die prematurely, that would be the maximum loss that could occur, but if the company had two such risks and both were to die in the same year, as is quite possible, the fluctuation

would be greatly increased. Therefore, a company is safer with one large amount than with two. In a similar way, it is safer with two than with three. And it is a fact, I think, that up to a fairly large number, every increase in the number of those large policies, instead of being an element of safety, would be an element of danger, which would have to be guarded against.

Then again, we must look at the question also from the practical standpoint, because we must remember that it is not always true that policies on separate lives are entirely detached from each other. If we were to have a half-dozen policies of large amount on persons closely related, or engaged in the same business enterprise, or crossing the Atlantic in the same steamer, it is very clear that the risk of having several of them die at the same time would be greatly increased.

Finally, I would remind our members that ten or twelve years ago this whole question was discussed in a paper by Mr. Clayton C. Hall, and if we wish to decide what the probable fluctuations will be by the method of least squares, we cannot do better than refer to Mr. Hall's paper, and the discussions thereon where the matter is gone into at some length.

Mr. McClintock: I agree in general with Mr. Nichols, Mr. Macaulay and others who have spoken in the same direction, and personally have little to say except to express doubts about the opinion that actuaries in general have heretofore been of the opinion that it is necessary to have a considerable number of maximum risks sufficient to make an average in order to be safe. I do not think that actuaries in general have had that opinion. The quotation from Dr. Sprague shows that it could not properly be said that that is the general opinion.

A good many years ago I had been invited to take a position in a pretty large company, and I had just assumed that position. I wrote a letter proposing that the maximum risk should be increased, for example, from twenty to twenty-five thousand dollars, and I entered into quite a little argument about it. I said something like this: "The amount of annual death loss is so many hundred thousand dollars in this company; the amount of annual fluctuation is a small percentage of that amount, and we at present have a good many policies of twenty thousand dollars; the additional risk if we increase the maximum is five thousand dollars on one risk. If we issue one policy the risk of loss on that policy is say one per cent., which shows a hundred chances to one that we don't have a five thousand dollar extra loss on that policy. If we have two policies it will be ten thousand to one against having both come in at the same time. If we issue three policies it is a practically infinitesimal chance that all three deaths will occur in one year, and so on. But if they did, it would be only fifteen thousand dollars in a total annual loss of so many hundred thousand dollars, the existing fluctuations being a great deal more." I cannot undertake to repeat the language, but that is the idea, and I

thought I made a pretty good argument. After the meeting of the trustees, I was told that they had voted down the proposition that I had made, being guided thereto by an argument of one of the other officers, who had said that it was perfectly well settled among actuaries that it was impossible to increase the risk unless you got a proper number of the increased maximum risks to make an average, and that it would be some years before we could possibly get enough of these twenty-five thousand dollar risks to make an average.

MR. WELLS: Mr. President, I read Mr. Nichols' paper, and I thought I agreed pretty thoroughly with what Mr. Nichols said, but I think I agree with what Mr. Macaulay has said also.

We have heard considerable about the average to-day. There is in Mr. Nichols' paper this language, being Mr. Nichols' statement of Mr. Young's proposition:

"That is, a company having decided upon a maximum line of three thousand dollars, and having all its policies within that limit, in order to increase that maximum line to ten thousand dollars or any other figure must have a reasonable expectation of securing enough of such ten thousand dollar policies to furnish an independent average."

Now, what do you mean by an "independent average?" I think the word "average" has been used here to mean a tabular death rate; the number that may be expected to die of those insured for a given amount; and it seems to me that it has been forgotten, except as Mr. Macaulay called attention to it, that the variation between the actual experience and the tabular number which may be expected to die will increase with the number of insured; as Mr. Macaulay said, there is more danger, there is mathematically a larger expectation of a variation if you have two persons insured than if you have one. If you have a thousand or ten thousand insured, your variation from the tabular expectation is likely to be much larger than if you had but one per cent. of that number insured.

While the probable number of deaths will approach more nearly to the tabular so far as percentage is concerned, that is to say, will differ from the tabular by a less percentage as you increase the number insured, so far as actual number or actual amount is concerned the probable variation from the tabular will increase as you increase the number insured; so that really a company that could stand a probable variation from the tabular loss if they had ten thousand policies of a given class could very much better afford to stand it if they had but a few policies of that class.

MR. NICHOLS: Mr. President, I do not know whether I can add very much to what has already been said in the article itself.

There seems to be on the part of some of the gentlemen a misapprehension, or not clear apprehension of what I mean when I say "the maximum line which any corporation can profitably write is largely determined" and I emphasize that word "largely" I do not

say it is entirely determined, "largely determined by the least number of policies into which its aggregate risks could be divided and still furnish a satisfactory average."

My meaning in that statement I think is indicated by what I say above, that if a company having for its maximum, policies of only five thousand dollars to a certain number, and from business considerations and other considerations decides or feels that it could safely divide that number of policies by ten of fifty thousand dollars each, and yet have a sufficient number for an average, then that would surely go to determine what would be the maximum line which it could write.

That illustration I put in another shape above, where I speak of the fifty thousand dollar risks divided into five thousand dollars each, and in reality, this illustration which I give seems to me to cover fairly well the chief point that I am undertaking to make in regard to this matter, that if there is a company with a certain number of policies of fifty thousand dollars each, entirely satisfactory from a business standpoint, I know of no business reason, I know of no mathematical reason why that company should be dissatisfied if it was able to divide each one, of the one or two or a half dozen of those fifty thousand dollar policies into five thousand dollar policies aggregating in each case fifty thousand dollars. That is my point.

In discussing this question there seems to be a natural tendency on the part of many minds, I think to rather beg the question. They start at the end where I undertake to leave off. They start with the assumption of a single risk, so large, in their minds, as to disturb the conditions of the company. That is the very question which was out of my mind. And the question is as to the ability of any one single risk, or any limited number of them, to create that disturbance.

The moment you assume as a matter of fact that one large risk or a few of them is going to disturb the existing conditions, it is entirely another matter.

Now, I think, the mind of an insurance officer naturally looks at the magnitude of the risks which he has on his books, and bases his assumptions on what would be a proper fluctuation from the size of those risks. In other words, if they are five thousand dollar risks only, these are his maximum. His mind naturally tends to argue that the maximum should be limited to such an amount as will not much exceed those five thousand dollar risks.

On the other hand, in a similar company which was dealing with fifty thousand dollar risks, their minds would naturally tend to argue that a fluctuation was not serious that would not greatly exceed the fifty thousand dollar risks.

Now, you see at once, that these are two positions which are antagonistic to each other, and yet which result from the limitation of the underwriter himself in judging simply from the character of his business what would be a proper fluctuation.

There is a tendency in all this thing, I think, for the mind to argue back to the limitation of insurance to special classes, and perhaps that

is in a measure the foundation of much of the strength of the argument in regard to single large risks being a disturbing factor unless enough of them can be secured.

My conception of insurance has been that so far as it properly can be carried out it should be dealt with on the basis of the insurance of the whole, by each other, rather than of special classes by themselves; that that is the underlying principle. There are certain business and other considerations which render it necessary in some cases to consider the insurance of special classes by themselves, but I have always regarded that as a subsidiary principle in the business, and that the broad fundamental principle should be so far as possible the insurance of all by each.

Now, I do not know, sir, that I have anything further to add to the remarks that have already been made. I think several of the gentlemen have sustained my position as stated, perhaps as well as I could sustain it myself.

NOTES ON A FACTOR, HITHERTO OVERLOOKED, OF THE RATE OF INTEREST—CHARLTON T. LEWIS.

VOL. VIII, PAGE 8.

WRITTEN DISCUSSION.

MR. C. C. HALL: It will not be our purpose to-day to discuss the paper presented by our greatly esteemed and beloved fellow-member, the late Dr. Charlton T. Lewis, on a Factor, hitherto overlooked, of the Rate of Interest, in any spirit of criticism, for the reason that its distinguished author is no longer with us to answer criticisms and close the discussion.

No one among us possessed greater learning, or greater power of analyzing an abstruse or philosophical subject and reducing it to its elementary principles than he. If there be anything in the way of criticism to say upon his presentation of the subject, it is not by way of correcting error, but only by way of supplementing his remarks, by supplying omissions in what was not fully expressed. In a brief paper it is impossible to exhaust a large subject. Upon one occasion, when Dr. Lewis had presented a paper upon a certain subject, I remarked to him afterwards in the intimacy of private conversation that I thought he had omitted some important factors. He pleasantly replied that he had only undertaken to make a brief address. If he had been writing a book on the subject, he would have included the views that I suggested.

In the paper we have now before us, he remarked that "A library has been written on the nature of interest, and the forces which determine its amount. The discussion is not closed, and the theory of the subject is still in confusion."

The comment which I would specially make upon this interesting paper is in reference to a particular point in which, it seems to me, he

failed clearly to express the conception which I am very sure was clearly apprehended by himself, and which he would more fully elucidate were he with us.

He remarked: "The level about which the rate of interest fluctuates, and to which, apart from disturbing influences, it always tends, is the rate of yield which enterprise expects from its use in industry. But this rate is always materially higher than the average rate of increase which industry actually obtains by its use."

He then proceeded to illustrate this statement by showing that the increase upon a sum of money during a period of years, at the rate of interest prevailing during that period, would be enormously greater than the general increase of capital, of wealth, during that period, and that, therefore, the prevailing rate of interest is and has been higher than the earning capacity of capital. The omitted factor here, it seems to me, is the allowance that should be made for consumption, for sustentation.

The earning of capital, the price paid for its use, does not go wholly to the increase of capital any more than the premium receipts of a life insurance company go *in toto* to the increase of its reserve or accumulated funds. Current risks and expenses must first be provided for. There must also be taken into account the actual destruction of property in war, by fire and flood, by disasters on land and sea, so that, therefore, from the increase of wealth, from interest, or the earning capacity of capital, must be deducted the proportion which has been destroyed, as well as that which has been consumed in the using.

Let us take a concrete example. Suppose a certain capitalist, having neither the disposition nor the ability to manage his property wisely and profitably, lends a portion of his capital to an enterpriser at three per cent. interest. The enterpriser employs it in various ventures, and as a result of his skill and energy earns six per cent. by its employment. Out of his earnings he has to pay interest for the use of the capital amounting to one-half the amount of his earnings or profits. So the capitalist and the enterpriser enjoy incomes of equal amount, the one from lending out the use of his capital, the other from the result of his skill in its employment. But suppose each of these men live up to the full amount of their incomes—that they pay out to the butcher, the baker, the tailor, the haberdasher, and the wine merchant, in the purchase of commodities which perish in the using, every cent of their incomes. Then at the end of the year, or the period under consideration, there is nothing left so far as they are concerned but the original capital. The rate of interest has been three per cent., but there has been no accretion of wealth; the earning capacity of capital has no more than kept pace with its waste, or to express it in less discouraging terms, its consumption. If we were talking of a railroad, we would say that the earnings had merely sufficed to meet the operating expenses and maintenance of way.

If under the assumed conditions there has been any accretion of wealth in the community, it is because the butcher, the baker, the

tailor, the haberdasher, and the wine merchant, with whom the assumed capitalist and enterpriser have dealt, have been wiser in their generation, and saved and put away a portion of their profits. So that the total savings become then a percentage only of the first earnings, or as we might say in mathematics, a sum of differences of the second order.

But the important point which Dr. Lewis sought to enforce, and the lesson which he sought to teach was, that the rate of interest was determined by psychical facts as much as by material conditions. While the price of any commodity, including under that term the use of money, is recognized as determined by the relations between supply and demand, the supply of money or capital offered for loans, or seeking investment, that is to say, on the market, is not measured by the amount of capital in existence, but by the amount which the holders are disposed to venture, withdrawing perhaps from the public funds or other permanent holdings for ventures which promise more lucrative returns.

Dr. Lewis expressed his conclusion thus: "The reason is that the demand is determined, not by the experience of past productiveness, but by the hope of future profits . . . . . This observation, as clearly entitled to be called an economic law as any generalization of its class, is a typical instance of the fundamental truth in economics that psychological conditions can never be neglected in estimating the effect of human actions on the production, distribution and consumption of wealth."

This statement I believe to be absolutely true. I would only conclude by placing in comparison with it the dictum of John Stuart Mill upon the same subject.

He wrote: "The rate of interest will be such as to equalize the demand for loans with the supply of them. It will be such, that exactly as much as some people are desirous to borrow at that rate, others shall be willing to lend. If there is more offered than demanded, interest will fall; if more is demanded than offered it will rise; and in both cases, to the point at which the equation of supply and demand is re-established.*"

MR. IRELAND: A reading of the paper that Mr. Lewis presented to the Society last May is a vivid reminder of his strength and energy as he then stood before us, and a striking evidence of our loss by his death, that then seemed so far away. He had not presented many formal papers at our meetings, but his contributions to our knowledge of insurance law, especially to the law of assignments, his vigorous denunciation of the theory that there could be such a thing as an individual reserve on a specified policy, and his confident and hopeful view of the future course of interest rates, are among the things that his hearers will distinctly remember.

In the paper under discussion, interesting throughout, there are passages here and there that are especially brilliant and characteristic.

* Principles of Political Economy; Book III, Chapter XXIII.

His definition of capital, and his discussion of the distinction between capital and property, illustrate this. Capital, he says, "is essentially a continuing claim upon or right in an ever-changing mass of things; it is a power, at any time, to appropriate a definite part of the wealth that is perpetually produced and consumed and reproduced by the whole community. Property is the physical product of labor upon material things, while capital is the creation of mind."

Political economy is as far as possible removed from the realm of exact science, and where one who is not an expert encounters expressions like these he is moved to say, what does this new form of words mean? Has our author got nearer to the truth than those who preceded him?

A little farther on we read: "Since people will always seek their own advantage, we quietly assume that they will act for that which *we know* or believe to be for their advantage. But the deviations from this rule . . . are far more numerous than the instances of it." Mr. Lewis might well have taken note of the fact that people are very far from always seeking their own great and ultimate advantage; in cases without number the gratification of some present wish leads them to do what they well know will result in their subsequent and greater disadvantage.

The special feature of the paper, the "factor hitherto overlooked," is summed up in these words: "The level about which the rate of interest fluctuates, and to which, apart from disturbing influences, it always tends, is the rate of yield which enterprise expects from its use in industry. But this rate is always materially higher than the average rate of increase which industry actually obtains by its use."

This way of looking at the subject from a psychological standpoint is characteristic of the whole paper. The phrase "capital is the creation of mind" has been already referred to; interest, we are told, "offers a problem, not so much in the nature and properties of things, as in the nature of men." The demand for capital "is determined, not by the experience of past productiveness, but by the hope of future profit."

One of the main arguments put forth in the paper to prove that the rate of yield which enterprise expects is always materially higher than the average rate of increase which industry actually obtains by the use of capital, is not, it must be confessed, a convincing one to the present writer. Mr. Lewis says, "It is four hundred and twelve years since the discovery of America. The average rate of interest on this continent throughout that time has been more than six per cent. But if one dollar had been saved each year, invested at that rate, and accumulated until now, the fund would amount in 1904 to four hundred and forty-seven thousands of millions of dollars; or nearly five times the aggregate estimated wealth of the inhabitants of America." Now, setting aside all query as to when the first saving and investing of dollars began in this country, it is not apparent how any information as to the rate of interest earned and paid on an invested dollar is

to be derived from the sum that is in the investor's hands at the end of a year, or a term of years. For by far the most common use that is made of an interest income is to spend either the whole or a considerable part of it, and the instances are few indeed in which cash earnings, either interest or profits, are wholly and continuously re-invested and become part of the original plant. These remarks of course do not apply to the increase of value of real estate. Mr. Lewis's theory is a very interesting one, however, and his references to the "plurality of failures," and the "wrecks and losses" by which the path of enterprise is strewn, are none too strong.

The whole paper gives a little hint of the very widespread learning of its brilliant author.

MR. WELCH: It was with a feeling almost akin to reverence with which, in compliance with your request, I undertook the task of commenting on the last paper written by Mr. Lewis, the delivery of which was almost his last public duty. Aside from the inability, of which I am conscious, to give the paper such criticism as it deserves, I am also weighted with the knowledge that the brain which was ever so keen in debate is now at rest, and the lips from which we all loved to hear that eloquence which few to-day command are closed forever; and yet, since my criticism is really but a repetition and an avowal of the beliefs he so tersely expressed, I feel glad in having this opportunity of simply calling attention to Mr. Lewis's paper. This question is in a measure a theoretical one, on which there may be many different opinions, and on which discussion is confined mostly to agreement with ideas already expressed or the substitution of other ideas equally as incapable of mathematical proof.

When Mr. Lewis delivered his address on "Interest" before the Fourth International Congress of Actuaries last year, he drew from some of the prominent Europeans present the criticism that he was "extremely radical and iconoclastic." I suspect, however, that Mr. Lewis has only gone a step further than they; has resolved the problem into a simpler equation; has found that what certain economists have called causes are but concomitant conditions arising from certain real causes which lay deeper, and that the necessary exceptions to the old rules which were made by all economists will be no exceptions when Mr. Lewis's law is applied.

In his preliminary discussion of property and capital, he differs little from other economists. Perhaps he goes a little further than some when he describes property as "the physical product of labor upon material things, while capital is the creation of mind. Property is the body, capital the soul of industrial life." But in the main there is little to criticise by any one until he recites what he calls "facts which seem paradoxical to public writers on finance." He says: "Thus it is certain that the increase of wealth does not necessarily lower the rates of interest; that there is not a progressive and secular diminution of these rates as the world grows richer and industry

more productive, and in particular the growth of the stock of gold and of its substitutes in use as money has no tendency to make the use of capital cheaper." In these statements Mr. Lewis agrees with certain writers and at the same time differs radically with others. He does not accept these conditions as fundamental causes for the described results. They are merely fortuitous conditions which do not affect the rate of interest, the true cause for change in that lying beyond, and this cause is best stated in his own words: "This theory" —the theory of "Interest" as explained in the new International Cyclopædia, which he gives in full—"is defective in taking no note of the vast difference between the productiveness actually experienced in the use of capital and the productiveness anticipated by the manufacturer or speculator in the use of the capital he needs. Economists have long perceived that periods of invention, discovery and enterprise are those in which the demand for capital is effective and interest high. . . ."

"The reason is that the demand is determined, not by the experience of past productiveness, but by the hope of future profit. Experience is one factor in inspiring hope, but by no means the most general or the most powerful. It is proverbial that the most sanguine adventurers in industry and speculation are not the most experienced in business. The excess in the great aggregate, of the expectation above realization is a constant fact in the modern world of production and trade; and since it is always the expectation, not the realization, that determines the demand for capital, the rate of interest, under all fluctuations, maintains a level materially higher than the average increase of capital."

Such conclusions does Mr. Lewis draw at the close of a paper full of interest from start to finish, and while many of his trains of reasoning present most interesting subjects for discussion on other lines, I wish to call attention especially to that part which perhaps to us is the most interesting, namely, his prediction of the future rate of interest.

He says, "In short, interest, like every other inquiry in economics, offers a problem not so much in the nature of properties of things as in the nature of men. . . . Habit, prejudice, imperfect knowledge, sudden impulse, waves of sentiment, sympathy with associates, sanguine hope, rash trust, or excessive timidity affect men's views and their interests, and thus modify all economic conduct and control large portions of it. If prudence were the supreme factor in practical life, which some economists assume it to be, the world would be richer, duller and less worth living in than Holland was a century ago.

"These considerations may satisfy us that interest is a fluctuating, not a gradually diminishing fact. History proves that the rate at which savings may confidently be accumulated varies under many influences, chiefly the varying energy of enterprise and speculation. . . . . That the latest of several well defined secular waves of change in the rate of interest covering the civilized world began to rise about 1845, and continued on the whole to advance until about

1872; that a retrogression then set in which lasted until five or six years ago; that the prevailing rates were then at the lowest, and have since slowly increased with impressive indications that the tendency will be upwards for many years. . . . ."

"But this expectation, it must be insisted, rests upon the nature of man in a progressive society and not upon the nature of property. Were the spirit of enterprise destroyed and the speculative hazards of fortune ended, the demand for capital in industry would be limited to the amounts needed under old and tried methods of production."

If we are willing to accept these conclusions as true, we have only to prove that we are living in "a progressive society" to feel reasonably sure of the increasing rate of interest which Mr. Lewis predicts. Ten years ago it might have seemed as if a generation which had seen the development of such marvels as the last half of the 19th century witnessed must be followed by one less productive of inventions which revolutionize the habits of life and employ vast sums of money in their development. Yet at the opening of the 20th century we see already the beginning of revolutions in locomotion on land and sea, while travel through the air is confidently predicted, and these, together with the transference of thought from continent to continent without the use of a material medium open before us, directly and indirectly, opportunities for investment and material development which the 19th century never had presented to it.

Of interest also on this point is the paper of Dr. L. Grossman, read before the Fourth International Congress of Actuaries, in which he stated his belief that the low water mark of interest has been reached.

In his article on "The Investments of Australasian Life Insurance Companies," recently delivered by John B. Gillison, F. F. A., F. I. A., before the Faculty of Actuaries, the author makes this simple allusion to the point under discussion:

"I need not refer at length to the steady fall in the rates in Australia as well as in Britain, for, as lenders in both countries are well aware, it now appears to have been arrested, and it is possible we may have a series of years of increasing rates."

In conclusion, I can but repeat my own concurrence in Mr. Lewis's views relative to the causes of changes in interest rates, although I do not follow him in all of his arguments in regard to the relative rates of "interest" and "profit"; and in pushing aside the material concomitants, I think Mr. Lewis has found true causes in psychic forces in which not only in this question of interest, but also in many of our own problems in life insurance will at last be found the true solutions.

AN EXPERIMENT WITH THE SPECIALIZED INVESTIGATION—RUFUS W. WEEKS.
VOL. VIII, PAGE 17.

WRITTEN DISCUSSION.

MR. VAN CISE: The Society is certainly under great obligation to Mr. Weeks for the information contained in his paper, and for the care and labor which were bestowed upon its preparation. "The Special-

ized Mortality Investigation" offers a large field for comment and discussion, but I shall not undertake anything further than a few random remarks in reference to the tables furnished by Mr. Weeks, and his deductions therefrom.

I certainly have no desire to criticise, but I think that Mr. Weeks was slightly in error in his declaration that every one of the seventy-six classes for which the figures are given in his paper "was supposed in advance to represent a blemish." I, of course, do not know the opinions of the individual members of the Committee in this respect, and they would doubtless have differed among themselves in their views as to the showing which would probably be made by a number of the classes; but I think all would have agreed that a few of the classes would probably indicate a favorable rate of mortality. I will mention two as examples:

Class 70 was composed of men of normal weight where both parents had reached the age of seventy-five. This class would naturally make a good showing. Class 6 was composed of those born in Sweden and Norway. As a general rule, the death rate among those born in Northern countries is more favorable than among those born in Southern localities, and according to my recollection, one of the Swedish delegates to our recent Actuarial Congress claimed that the rate of mortality in Sweden and Norway was less than in any other European country.

I might mention some of the other twenty-seven classes selected by Mr. Weeks in which an unfavorable showing might not have been reasonably expected, but it is sufficient to say that, in my judgment, nearly all the classes derived such benefits from the effects of selection that the twenty-seven classes selected by Mr. Weeks show a considerably lower death rate than would have been shown by the total business of the companies, had this been made up under the supervision of the Committee.

It is, of course, somewhat remarkable that the aggregate deaths of the entire seventy-six classes should be slightly less than the aggregate number expected according to the assumed standard, but I must attribute this to the benefits of selection rather than to any mistake made by the Committee as regards the standard fixed by them to represent the probable death rate among assured lives, taken as a whole, in American companies.

Nevertheless, while for the reasons stated I believe that the twenty-seven classes selected by Mr. Weeks as a basis for his tables show a lower death rate than would be shown on the total business accepted by the companies, yet this does not lessen the value of the figures obtained, or the comparisons derived from these figures, which show the progress of the death rate as between actual and expected during the years in which the assurance was in force.

The percentages of actual to expected deaths clearly indicate that the benefits of selection do not reach beyond the end of the fourth year, although it has been a popular idea that these benefits were also

shown during the fifth year. No doubt, in American companies, the high lapse rate after the payment of only one premium, has a tendency to shorten the period during which a benefit is derived from selection, and this may also be shortened because of the considerable number of policies given up as soon as surrender can be made for cash or paid-up assurance. The tables certainly indicate that the death rate during the fifth and subsequent years remains substantially the same until the tenth year, when there is a marked increase, which again occurs after the fifteenth year. I agree with Mr. Weeks that the increase at these two periods is doubtless attributable to the maturing of Ten and Fifteen Year Endowment policies, and with them might be included Deferred Dividend Life policies, whose holders have the privilege of withdrawing the entire reserve as a cash value at the end of ten or fifteen years.

In speaking of the percentages of actual to expected deaths derived from the experience of twenty-seven classes, Mr. Weeks says: "We should interpret these figures as giving a fair picture of the normal Endowment mortality of the combined companies," and I agree with him in this interpretation rather than in his previous reference to the death rate among these twenty-seven classes, when he made the statement that " it is not likely that any large section of the impaired or classified business would show a lighter mortality than the total 'accepted' business of the various companies."

It is generally believed, and experience has corroborated this belief, that the death rate on Endowment policies is considerably lower than the death rate on Life policies, because of the element of selection which seems to operate in favor of those who pay the higher rates of premium as against those who prefer contracts at the lowest possible cost.

A singular fact shown by the tables obtained by Mr. Weeks, is the decrease in the death rate after the twentieth year as compared to the rate experienced during the five preceding years. I do not pretend to account for this decrease, but it may be owing partly to what might be called " the survival of the fittest," and to the fact that after the twentieth year there are but few survivors of those who entered at advanced ages. I am inclined to believe that the experience of nearly all companies indicates a high death rate among old men who assure their lives, because of a certain amount of selection against the companies among this class which cannot be prevented even by careful medical examination.

I fully agree with Mr. Weeks in the caution which he calls for at the close of his paper, as to our keeping the fact in mind that the "Specialized Investigation" deals only with lives assured, and not with the amounts assured. The results shown by the experience in Class 1 in which the amounts were $20,000, or over, on the same application, clearly tends to prove that the death rate in respect to amounts assured is likely to be worse than the experience on lives, whatever may be the character of the groups under observation.

MR. WELLS: The mere statement of exposures and deaths contained in the Specialized Mortality Investigation would be of no use for the purposes for which the investigation was undertaken if there were no means of comparing it with the experience of the companies as a whole, or of individual companies. If the Committee had not made a comparison of the actual deaths with the expected, as computed by some standard, each person desiring to profit by the experience would have been compelled to make a comparison for himself. Doubtless, as Mr. Weeks states, the ideal standard for comparison of the aggregate experience under the classes given would have been the entire experience of the companies under the policies issued during the thirty years included in the observations. Such a comparison was impracticable. The Committee used, of necessity, an artificial standard corresponding to the impressions of its members as to what the normal mortality rates among the average of American insured lives have been. The question as to whether the standard assumed was a proper one at once presents itself.

Probably many besides Mr. Weeks and the writer were struck by the fact that, notwithstanding the classes in question, almost without exception, were under suspicion, many of them showed a mortality better than the assumed standard; and taken as a whole, the deaths were slightly less than the expected deaths. But our surprise, I think, merely showed that we had not given sufficient heed to the warning of the Committee, which Mr. Weeks repeats; that the experience covers only selected risks of the various classes, and not the generality of people belonging to the classes. If the assumed standard was a fair one, and the mortality of a given class was found to correspond with such assumed standard, it would merely mean that the companies had been successful in the attempt to select from that particular class of persons those who were, upon the whole, good average risks for insurance. If the mortality for any given class of the insured were better than the assumed standard, it would show that the companies had been more rigid than was necessary in their selection from that class of persons. If the mortality among those insured in any given class exceeded the expected, it would show that the companies had not been sufficiently rigid in the selection of persons of that class. And whether the assumed standard be in all respects correct or otherwise, the experience does undoubtedly show that the companies have given much more weight to the assumed blemish in certain classes than they have to that in certain other classes, as compared with what the facts, as disclosed by the investigation, warrant.

The peculiar gradation of the percentage of actual to tabular mortality by the American Table for twenty-seven classes, as shown by Mr. Weeks, I am wholly unable to account for. It seems to me that it must be purely accidental. We cannot suppose that Mr. Weeks connects the mortality in these classes in any way with endowment mortality. Indeed, such a result in case of endowment insurance could only be produced by a very considerable percentage of policies written upon the shorter terms, and a very considerably lighter mortality in case of such shorter term policies.

Column (5) of Mr. Weeks's last tables, for Classes Nos. 4, 12 and 31, suggests a possible error in the allowance for selection made by the Committee. For further investigation of this matter, and also for comparison with Mr. Weeks's table showing for twenty-seven classes the "actual deaths by insurance years compared with expected according to the American Table," I have prepared for Classes 4, 69, 70 and 72 combined, the following table. These classes were selected because they show the largest number of deaths of any of the classes given, and with a single exception they show the largest number of exposures in the first year of insurance. The actual deaths were 53,244, and the expected deaths by the Committee's standard were 53,897, making the actual deaths 98.8 per cent. of the expected. The exposures in the first year of insurance amounted to 742,479. Two of these classes, 70 and 72, are included in the twenty-seven classes of Mr. Weeks, and the other two, 4 and 69, are not so included.

This table suggests that, compared with the mortality of these classes, the table assumed by the Committee, the Modified Healthy English Table, shows too high a rate of mortality at the younger ages, and perhaps somewhat too low a rate at the older ages; also that the Committee made too large a deduction in the first two years of insurance for the effects of selection, and that a gradually decreasing effect remains for many years.

CLASSES 4–69–70–72 COMBINED.

Percentage of Actual Deaths by Insurance Years and Groups of Ages at Entry, to the Tabular, according to the "Modified Healthy English Table."

Percentage of Actual to Tabular for Ages at Entry.

Insurance Years.	15–28	29–42	43–56	57–70	15– 0
1	50.9	53.2	62.8	73.0	57.6
2	70.1	72.6	86.0	94.8	78.7
3	73.1	76.3	87.9	92.0	81.3
4	75.6	84.9	90.0	86.0	85.7
5	83.3	84.5	97.0	91.7	89.5
6	78.5	82.8	93.3	106.4	88.4
7	76.7	88.7	93.0	102.8	90.5
8	85.2	88.4	98.1	92.3	92.2
9	84.0	85.9	93.8	108.8	91.1
10	78.6	89.3	96.1	117.2	93.9
11	81.3	98.0	97.4	112.0	97.7
12	97.5	94.9	103.5	104.8	99.6
13	89.3	93.4	107.5	97.3	99.4
14	77.1	100.8	108.7	87.6	101.4
15	92.8	96.2	103.0	93.0	98.7
6–10	80.4	86.8	94.8	105.1	91.0
11–15	87.4	96.7	103.8	100.1	99.3
16–20	94.3	106.7	110.2	96.7	106.7
21–30	100.0	102.0	102.7	93.6	101.8
6–30	86.4	95.0	101.3	101.4	97.5

WRITTEN DISCUSSION.

MR. NICHOLS: The interesting and instructive paper of Mr. Henderson is well qualified to enlarge the conceptions of the ordinary student of Actuarial Science. It introduces him to a domain of which his own special studies are but a limited branch. The mathematical doctrine of probabilities deals chiefly with a special class of causes known as accidental or fortuitous, in which every effect is regarded as the result of innumerable small causes equally likely, and operating on the principles of the well-known law of error. In practical statistics a different class of causes are met. The forces of nature and the sociological laws which underly the subjects chiefly dealt with by the statistician, are directed largely by causes which are either constant or variable or periodic in their character and unequal in their efficiency or weights. Though in a broader generalization they might be viewed as phases of accidental causes, within the limitations of the practical statistician they must be dealt with as of an essentially different character. Hence the mathematical formulas so familiar in the law of error have but a limited application. In those related phenomena which are the subjects of ordinary statistical inquiry, the possible combinations of such causes are innumerable and within the limitations imposed by physical or social laws, the equations of statistical curves may take on a wide variety of forms, of which the law of error is as Mr. Henderson points out, only a special case of a single class, In fact, it may be regarded as the form assumed by the statistical curve, when all other than purely accidental errors of this particular kind have been eliminated.

Mr. Henderson starts with a discussion of the general elements common to all these curves or rather of the related functions when their relations are such that they can be graphically expressed in the form of a curve. The important question to the statistician is, how the curve with which he is dealing can best be made to tell its story and answer the queries which are the subjects of his investigation. The means and the limits of such statistical functions, have, from the earliest dawn of history, been commonly recognized by mankind as fundamental subjects of inquiry in such connection. Averages and extremes, the one fixing the limits and the other the general weight or effect of the observations as a whole, are of universal recognition, and the very starting point in any attempted comparison of statistical curves.

Mr. Henderson first deals with these fundamental features. But the objects of statistical inquiry are by no means satisfied by the determination of these two functions. Others are required which vary with the nature of the question, and Mr. Henderson proceeds to treat of a few of those which are more generally recognized. Among these

are the median and the mode. These too are in a crude way recognized by mankind as important functions. Every shrewd business man naturally asks regarding an uncertain venture as to the most probable result of his experiment and the probable loss which he must expect to stand in the event of its failure.

The bearing of the higher moments on statistical inquiries is not perhaps so obvious. Why should the square of the departure rather than the departure itself enter into inquiry? An apt illustration of certain classes of questions involving this function is furnished in testing the skill of the marksman. As the departure from the bull's eye increases, the area of the circle within which his bullet may strike enlarges, and the lack of skill is not measured by the mere lineal departure, but by that departure multiplied into the circle itself, in other words, the measure of his skill is the inverse of the square of the departure which is the second moment. On the other hand, were he a bowler rolling a ball on a plain surface, the mean departure itself, or the standard deviation mentioned by Mr. Henderson, which is a lineal measure would be the appropriate test of skill. Similarly, in the frequency curve which represents the rate of mortality, every actuary is familiar with the uses which are made of the different functions of that curve.

Mr. Henderson next proceeds to discuss the functions represented by the third moment and those of higher orders. It will be sufficiently obvious, from what has been said, that the comparisons of these moments, as pointed out by Mr. Henderson, are in reality so many different tests of the character of the dispersion of the various values of the variable, or of the manner in which those values are grouped in relation to the mean and to each other. Such inquiries have a most important bearing on many statistical investigations. Meteorological inquiries furnish an apt illustration of these. The skewness of the statistical curve is apt to be a characteristic of the related functions dealt with in ordinary statistics, and indeed may be said to be a characteristic of the workings of most natural forces. It is seen in the mortality tables where a slowly increasing death rate at the younger ages is followed by a rapid increase of mortality in later years. It is specially pronounced in meteorological phenomena and those which are related to them. The distributions of temperature about the normals at the various seasons are exceedingly unequal as are apt to be the distributions of rainfall and moisture and sunshine. These features in their combination determine the climate. The influence of that climate determines the character of the crops. We may note the sporadic blossoms which precede the sudden bursting into bloom of a vegetable species. We may note the sudden and premature return of summer weather in the spring or of winter weather in the early fall to be followed by the normal conditions of the advancing season, or the sudden and violent fall or rise of temperature which we term a hot or cold wave, followed by the gradual recovery of normal conditions. In the death rates we may

observe the sudden rise attendant on epidemics or unfavorable meteorological conditions followed by the gradual return to normal, while a corresponding abnormal reduction of the death rate is rare. This is the skewness of the ordinary death rates of our cities. Similar laws are observable in sickness. In the field of economics we meet with financial panics, with their slow recovery. In the field of chemistry we see the gradual approach of the solid to the liquid state followed by its sudden change to the latter when the critical point is reached and the sudden and violent change of the latter again to a gas when its critical point too is reached. The water contracts as its temperature falls to the critical degree, then again expands until congelation, where it reaches its inferior maximum. The electric tension insensibly grows in the atmosphere until the force of the lightning suddenly restores the equilibrium. These are all illustrations of that skewness and dispersion or otherwise of the ordinates to which Mr. Henderson refers, and which is so universal in related natural phenomena. I dwell on them for a special reason. Their explanation largely lies in the periodic and fluctuating character of the causes which produce them, and the interposition of constant causes as well. If in place of the urn with its balls, which we interrogate to learn the mathematical laws which regulate the operations of our so-called accidental causes, we substitute a series of urns from which the drawings may be made, in each of which the relative number of white and black balls are known and are different, and again assume a constant definite tendency of the drawer to select a white ball rather than a black, because of some peculiarity in its feeling, we have an illustration of the combination of causes so generally met with in the operation of physical and social forces. The selection of the urn is determined by purely accidental causes. The character of its contents will vary according to the urn, while a constant cause is operating to favor the selection of a white ball regardless of the urn. It is easy to see that the curve of probability would be represented by an equation widely different in its character from that so familiar in the ordinary doctrine of chances. Strictly speaking, while the operation of periodic and fluctuating causes, as well as those which are constant, may be said to be regulated by definite laws, the same is not true of those recognized as fortuitous. There is no probability in favor of or against an accidental cause.

Now the special point which I wish to make in connection with all this, is that we may find in this distinction between the causes which operate in the actual world around us, and which are the subjects of statistics and those which are assumed in the mathematical theory of probability, a partial solution of one of the gravest difficulties attached to certain phases of the doctrine of evolution. How is it possible, has been asked, that the orderly and complex system of the universe or of the world around us with its teaming organisms could be the outgrowth of any fortuitous concourse of atoms? The mathematical doctrine of chances would seem to exclude the possibility. I

seem to find a partial answer in the fact that the causes which prevail in the material world are essentially different in their character and mode of operation from those that are associated with the ordinary doctrine of chances. In the very skewness and variant concentrations and diffusions of their resultant effects, they tend to produce that heterogeniety which is the first step in evolution. But I have no time to press this point, further suffice it to say I am dealing purely with the scientific and not with the teleological or religious aspects of the problem.

The important relations sustained by the doctrine of moments in the discussion of frequency curves, is shown in the investigations of the ordinary curve of probability where the fundamental moment, as in all frequency curves, is the mean or centre of area while the second moment, involving the squares of the departures, is the foundation of the doctrine of least squares, by which not only the probability of error is determined from the well-known probability integral, but by which the most probable mean is computed in observations of unequal weight. But the questions involved in that curve chiefly relate, as is well known to powers, no higher than the second. In general statistics we meet with a broader field and the operation of causes so modified in their character and effect that it is easy to see why higher moments may be important for comparison, and it may be noted in the paper of Mr. Henderson, how in the comparison of moments of different orders, they are first raised or reduced to the same exponential rank in order to be comparable. The important function, Beta prime, which he discusses and which is the quotient of the square of the third moment by the cube of the second, is in effect a comparison of sixth powers.

Mr. Henderson discusses at considerable length the theory of a special family of curves whose principal claim to notice arises from the fact that many of the distributions occurring in nature resemble some of its types and which includes, as one of its members, the curve of frequency of the probability of an event which has been observed to happen a specified number of times. It is obvious that we have in the latter an analogue of the causes operating in nature. When an event has been observed to happen a specific number of times, there is the presumption of definite causes operating to produce it, combined with those which are fortuitous and unknown. The class of curves which include the former would naturally include the latter. The general equation of the family itself, as given by Mr. Henderson, suggests that peculiar combination of constants and variables so familiar in Gompertz and Makeham's law of mortality and which the former attempted to explain by the operation of analogous physical forces. Where the constants in the equation are reduced to an expression of the precision of the curve and accidental causes are left to shape its general form, as shown by Mr. Henderson, we have the theoretical law of error.

In dealing with statistical questions, it must be borne in mind that we are dealing primarily simply with the observations themselves. as related functions from which the form of the curve which joins them is to be judged, or to which if a special form is for any reason to be preferred it is to be fitted, and Mr. Henderson in conclusion shows the method by which this may be done through the use of the various moments, and the conditions which limit their application. The explanation of this in a non-mathematical way is clear when it is considered that each separate departure enters into the formula by which the moment of the whole is computed and consequently the latter changes with the degree to which those departures are raised. In using these moments we are seeking the best values of the constants, just as in the application of Makeham's formula we seek such values by framing equations from different combinations of the observations.

I have not, it will be observed, attempted any criticism of Mr. Henderson's methods of treatment in his admirable paper and without a more detailed study of the subject, should hardly be qualified to attempt it, if any such criticism were needed. His processes speak for themselves to those who are competent to follow the reasoning. He has carried us into an enlarged field of actuarial inquiry and I have sought to indicate in a general way the relations which subsist between the ordinary investigations of the actuary and that broader science of statistics, of which they are but a single phase, as well as some of the fundamental features of that science itself.

Oral Discussion.

Mr. Henderson: I simply wish to acknowledge to Mr. Nichols the kindness with which he has reviewed this paper, and to point out that those types of frequency curves which are contained in this last table contain, some of them, very many sub-types, which would offer a very interesting field of investigation for any of our junior members who are interested in that way, by the variation of the different constants involved in the equation. In particular I would simply like to point out that in type III, letting n in the formula approach gradually to its limiting value of zero, the curve has already become infinite at the two limits and is hollowed at the centre and gradually approaches the base, gradually squeezing the area of the curve out to the two ends, and approaches the ordinary case of probability, where p is the chance of one extreme value and q that of the other.

WRITTEN DISCUSSION.

MR. HANN: Mr. Teece has furnished the Society with a valuable contribution on the above. It was inspired by the appointment of a Royal Commission to consider the question after Mr. Coghlan, the Government Statistician, had written a pamphlet on the subject.

Mr. Teece's paper is concise and to the point in dealing with measurable quantities from the most authentic sources.

He starts off with certain hypotheses, generally conceded by political economists to be beyond dispute. He points out that the most marked decline followed the financial crisis of 1893, accompanied with a prolonged drought which has but recently disappeared. Table I proves married males to male population of the same ages between

20 and 30 had declined 12½ per cent. in 20 years.
30 and 40 had remained almost constant.
40 and 60 had increased nearly 8 per cent.

This decrease in the proportion of young married men, and appreciable increase among the older, make for a decline in the birth rate.

Table II is derived from the four last censuses.

A summary therefrom gives four quinquenniums from 1880 to 1900, proving that the proportion of males to females has steadily decreased. This is the natural outcome of a young country as it becomes more settled and under *prosperous* times should aid the birth-rate.

The marriages to total mean population for twenty-one years ended 1902 have decreased eight and a half per cent.

Married Women. By the census of 1901, the proportion of married over thirty-four years of age exceeds that below thirty-five by more than thirty per cent., which coincides with the rapid decline in birth-rate, and the fact is established by Mr. Teece, from extensive figures, that there are more old and fewer young people than formerly, which again makes for a decreased birth-rate.

On page fifty-seven a slight error has crept in under "Women Returned as Married," 1881 census. Under age thirty-five the figures should be increased six, and diminished by six for those thirty-five and over.

Table III is full of interest and deals with the population for twenty-one years ended 1900. The salient points brought out are:

The birth-rate of mean population has fallen in twenty years over twenty-six per cent.

The birth-rate of female population has fallen nearly twenty-nine per cent.

The birth-rate of females between fifteen and fifty to total female population has fallen nearly thirty-three per cent.

I find the average of female population from fifteen to fifty to be a shade over *one-half* of the total female population for the twenty years ended 1900.

As a proof of the uniform action of the human will the proportion of the married out of the female population has practically remained constant for twenty-one years.

The married rate, however, among females from fifteen to fifty has decreased fully twenty-four and one-half per cent., while the proportion of births among married females, from fifteen to fifty, has decreased nearly twenty-nine and one-half per cent., or measured by quinquennial groups, twenty-seven per cent.

It has been proved that married females, to total females, have remained constant throughout the period ; that the rate of married females has fallen only three and one-half per cent.; nevertheless, the birth-rate has fallen from 173.93 per thousand, for the earlier period, to 116.87 for the later, a decline of nearly thirty-three per cent. It is true that the decline is largely due to the fact that the proportion both of married men and married women, at the ages at which generation is most active, is largely decreased, attributable, no doubt, first, to the "prudential check," that comes into play on the part of a large class which seeks to maintain its social position in the face of adversity; and secondly, to unnatural and unworthy customs, that, if persisted in, must not only seriously diminish the rate, but is bound to lead eventually to a weak and puny race. It is hoped that more prosperous days will mitigate the prevailing evil custom. Nothwithstanding the extragavance induced by love of show, etc., we trust that it will ever remain true that—

"The hand that rocks the cradle rules the world."

Other causes might be dwelt on as factors retarding the natural growth of New South Wales, such as the great increase of migrants from the country into cities. The discouragement of immigration through the labor unions. This is largely a pastoral and agricultural country. It needs sorely a constant stream of enterprising immigrants, especially if they have a little capital. The competition between Revenue and Expenditure is keen; the latter exceeded the former last year. Finally, the coercive legislation of the laboring classes has added to the trouble, and greatly retarded the growth of New South Wales, which comprises thirty-six per cent. of the whole population of the commonwealth.

Mr. W. C. Wright: I can find nothing to criticise in the form and thoroughness with which Mr. Teece has treated the statistical data on which his paper on the Decline of the Birth Rate in New South Wales is founded. I do not question the reliability of the facts he presents. But I see matter for encouragement rather than for alarm in these facts. They appear to me to mark the progress of refinement, forethought, ambition, and civilization. The hands

which rocked the cradle, I suspect, have more to do with this change than any others. No matter how zealous we may be to popularize and defend the beneficence of the institution of life insurance, we surely are not, or should not be anxious to multiply crowded tenements, or East Ends, and Boweries, for the sake of having more children to insure. Nor should we plan Johnstown catastrophes, and Hellgate horrors for the aid of their undoubted advertising power. Nor should it be a source of regret to parents to have gained two years of the society of their children.

Mr. Teece does not claim that the population of New South Wales has decreased, though his figures do show that the rate of increase has fallen from the very high rate of 50.6 per cent. between 1881 and 1891 to 20.1 percentage between 1891 and 1901. I do not believe there is the slightest danger of a diminution of this rate to the vanishing point, for any length of time, wherever the political and other conditions of life are such as to insure comfortable support to the existing population. The order of comfort generally aspired to by the body of the people cannot be too high.

There is a prevailing impression that among English-speaking communities at least, the physical condition of women has improved, and their stature has increased, and it is notable that if this is true it has occurred co-incidently with the change commented on by Mr. Teece.

Mr. Macaulay: The subject discussed by Mr. Teece is one which appeals not merely to actuaries and statisticians, but to statesmen and political economists in many parts of the world.

Dealing with the population of New South Wales, the facts which stand out upon the surface are, that while for the period of six years from 1880 to 1885 inclusive, the average number of births for each thousand of mean population was 38.52, the corresponding figure for the years 1896 to 1900 inclusive was but 28.44, while for the year 1900 alone it was but 27.72. In like manner the average number of births for each thousand of female population, fell by successive steps from 84.83 for the years 1880-85, to 60.34 for the years 1896-1900, and to 58.41 for the year 1900 alone.

Taking next the average number of births for each thousand of female population, including only those aged fifteen to fifty, a reduction is again noted from 173.93 for the years 1880-85 to 116.87 for the years 1896 to 1900, and 112.15 for 1900 alone. These statistics are sufficiently striking and leave the impression that there has been a marked decline in the birth rate. The problem which Mr. Teece discusses is whether there has been an actual decrease in the birth rate as these figures would appear to show ; if so whether it has been as great as supposed, and to what cause or causes it should be attributed.

In his efforts to get at the real facts, Mr. Teece has worked out many percentages and comparisons, by which we are enabled to obtain a partial answer at least to these questions. In investigating the sub-

ject one of the first points to which attention has to be drawn is, that there has been a considerable decrease, particularly during the ten years from 1890 to 1900, in the proportion which the males of ages twenty to thirty bear to the total population, the figures being 19.21 for 1890 and 16.75 for 1900. On the other hand, during the same ten years the proportion of males of ages thirty to sixty increased from 31.08 to 32.10. It is evident, therefore, that any conclusion based upon the numbers of males aged twenty to sixty, taking them as undivided groups, must of necessity be more or less erroneous, so far as a comparison between 1890 and 1900 is concerned for the latter group contains a smaller proportion of young men and a larger proportion of old than the former and a lower birth rate is therefore no more than normal. The principal reason for there being now a smaller proportion of young men in the colony than formerly, is no doubt to be found in the almost entire cessation of immigration to which Mr. Teece has referred. Whether there has been any emigration from the Colony I do not know. Cause and effect also of course react on each other, for any falling off at any time, in the number of births from this or any other cause results in its turn in the reduction in the number of young men twenty years later.

Let us now, however, consider the married groups. If we compare the married males aged from twenty to sixty, with the total males of the same ages, we find that the percentage remains fairly stationary at all three census dates, but with a slight tendency to increase. From this it might be supposed that the percentage of married males to the total males of the same age, has not materially changed from one census to another for any of the ages between twenty and sixty. Here again, however, Mr. Teece shows that the facts are not what this superficial examination would have led us to expect. The percentages of married males of ages twenty to thirty to total males of the same ages, show a marked and steady decrease, being 26.7 in 1881, 25.6 in 1891 and 23.5 in 1901. The corresponding percentages for males from thirty to forty show no decrease, while those for ages forty to sixty show a marked increase, being 64.6 in 1881 and 69.6 in 1901. Mr. Teece has therefore shown not only that the group of male lives between twenty and sixty as given in the last enumeration, contains fewer young men than were reported at previous censuses, but also that of those young men a smaller percentage are married than formerly.

In the case of women a very similar state of affairs exists. The percentage which the married women between ages fifteen and thirty-five bear to the total married women, decreased from 51.0 in 1891 to 43.4 in 1901, while the corresponding percentage for older women (ages thirty-five and over), increased from 49.0 in 1891 to 56.6 in 1901. Of married women between ages fifteen and thirty-five there was an actual decrease during the ten years, notwithstanding the increase in the total population. The average age of the married woman is thus considerably greater than it was ten years ago. Moreover, the number of marriages per thousand of females of ages fifteen to fifty has markedly

fallen away, being 38.40 for the six years 1880-85, 33.63 for the years 1886-90 ; 29.06 for 1891-95 and 28.95 for 1896 to 1900. This low marriage rate is perhaps more strikingly brought out by the fact that the census of 1880 showed 70,972 unmarried women between the ages of fifteen to fifty, while in that of 1900 the figure had swollen to 165,353, an increase of 94,381 in the twenty years, or 133 per cent., while the total population had only increased 84 per cent.

From Mr. Teece's statistics certain facts are clear:

(1.) Conclusions based upon comparisons of the numbers of births to total population, or even to total married population, or even still to total married population including only males between twenty and sixty, and females between fifteen and fifty, are by no means necessarily reliable, and in the case of New South Wales are certainly unreliable.

(2.) There has been an actual decrease in the marriage rate in New South Wales, and from this we are no doubt justified in concluding that full statistics would show an undisputable reduction in the birth rate when compared with the population, even if full weight be given to the question of age. This low marriage rate has been no doubt due to a considerable extent to the severe and prolonged commercial depression and should improve with the return of prosperity.

(3.) The statistics before us yield no positive answer to the question whether there has or has not been a decrease in the average number of children per marriage. A comparison of the percentages may, however, lead us to think that there have been other factors at work in addition to the two above mentioned. The reduction in the proportion of young people, and the decrease in the marriage rate, although such other factors may not have had as serious effects as is commonly supposed.

A practical evidence of the effect which the proportion of young people has upon the birth rate of any community is to be seen in the birth rates of Ontario and Manitoba. I have not the statistics by me, but I know that the fall in the rate in Ontario has been giving much concern, while on the other hand, Winnipeg is spoken of as a city noted for the great number of its young children. The young men and young women of Ontario migrated to Manitoba with the result that the birth rates of the new cities is decidedly high while that of the old cities has been lowered.

General population statistics are notoriously subject to misinterpretation, and Mr. Teece has certainly done well in pointing out the real nature of the problems involved in a discussion of the birth rate, not merely of New South Wales, but of any other country. His work, in fact, is a practical application of the motto of our society "substituting facts for theories and demonstrations for impressions."

Mr. Macaulay continued: Of course, the real problem that Mr. Teece and others in Australia are investigating is whether or not what President Roosevelt calls "race suicide" is being practiced. The importance of that question is greater than may at first be supposed, for usually the lower classes of immigrants to countries like the United States and Canada do not show any reduction in the birth rate, while generally speaking, it is the best classes in our communities that do show such reduction. It is, therefore, an exceedingly serious problem.

Some of the regulations which formerly prevailed in Quebec under the French *régime* may not be without interest in this connection.

In the early days of Canada the need of settlers was very great. The infant colony was waging an incessant struggle for its very existence with the English settlements to the South, and with the Indians to the West. They kept appealing to France for more soldiers and more settlers, but Louis XIV could not spare them. He needed all the men for his wars. Colbert, I remember, wrote out to the Governor of New France on one occasion and told him frankly that they could not send more settlers, and that he must impress upon the colonists that prosperity could never come to them except through the abundance of men, and that they must look to internal growth for that abundance, and not to France.

The result was that special regulations were adopted, having in view the increase of the population of the colony from within. Every arrangement that could be devised was adopted to increase the number of marriages, to lower the age of marriage, and to increase the size of the families.

Many of those details are now very interesting reading. For example, every settler who had an unmarried daughter, aged twelve or over, I think, had to appear once a year before the local governor and give a reason why she was not married. If the reason was not satisfactory, he was fined, and at the end of every six months thereafter he had to appear again and go through the same ordeal until the desired marriage took place.

The governor of the colony also had to make reports to the authorities in France as to what progress was being made along these lines. He had to give detailed statements with regard to the marriageable condition of all the officers in the colony. In one of these reports the governor went on to say that so many officers had been married during the year, and that three of the others, whose names he gave, were "almost engaged," and were having *pourparlers* with their ladies!

There was one radical difference between the nature of the colonization of New France and that of New England; the colonists of New England came chiefly in families, while those of New France were principally unmarried men. The large proportion of men was accentuated by the fact that the French government, in order to

colonize the country, disbanded one or two regiments. There were, therefore, considerably more men than women, and the demand for wives was great. In one instance, report tells us that the Governor of Three Rivers having died leaving a young widow, the demand for her was so great that she was married again actually before her first husband was buried.

In view of the insufficiency of the supply, the French government was asked to send out shiploads of women. The authorities at first went to the poor houses of Paris and Northwestern France, and selected healthy young women that were poor but of good character, and sent them out. Shortly afterward it was decided that country girls were best, and thereafter a propaganda was carried on throughout Normandy and Brittany, by which small bonuses were given to healthy country girls to induce them to consent to go to New France, and shiploads of them were sent out every spring for a number of years in the charge of nuns. A French writer named La Hontan gives a very amusing account of what happened at that time. About two weeks before the expected arrival of the ship from France, an order was sent around that every unmarried man in the colony must get married within two weeks after the arrival of the vessel. When the ship arrived in Quebec, we are told that the nuns divided the girls into three classes, according to their education and social standing, and these were placed in different rooms. The prospective suitor would come before the nuns, who would question him about his financial standing, etc., and would make up their minds as to which of these three classes they would assign him to. He would then go into the room where the ladies were waiting for him; he would question them, and they would question him. We are told that the first questions the girls asked were usually about his farm, where he lived, and details of that kind. The girls had the right to refuse any person. They were picked up very fast. We are told that the plump ones usually went first, because it was supposed that they would be able to stand the cold better than the thin ones, and that, moreover, they would not be so apt to travel around to their neighbors and gossip.

There were, says La Hontan, wherewith to satisfy the most fastidious. There were stout, and thin; tall, and short; fair, and dark; and in short, everybody could find one to suit him; and in an exceedingly short time the selections were made, and with the help of a notary and a priest the marriage was completed, and they went off to the farmer's house. The government supplied them at once with a small sum of money and a barrel of salt pork, a pig, and I think, a cow, besides other helps to housekeeping. As La Hontan says, "In this way Hymen, if not Cupid, was lashed into a frenzy of activity throughout the length and breadth of the land."

But officers needed wives as well as soldiers and peasants, and the records show that the authorities sent requisitions for a limited number of "select demoiselles" who could become the wives of officers. The report of next year says that too many had been sent; not to

send any more of that class, as the supply already exceeded the demand.

The reports of the next few years stated that there had been a tremendous number of births throughout the country. Not satisfied with even that success, however, bonuses were then given to increase the size of families. Every father who could show that he had twelve living children, not including nuns or priests—they excluded them—received a special bonus each year, as long as he had that number living. As a result of these and other regulations, a special character was imparted to the people, and we now have in the Province of Quebec a race, which although descended from that of France, are probably the most prolific on the face of the earth, without any exception.

Some little time ago, as a matter of common interest I worked out the average number of brothers and sisters given in five hundred life applications from French Canadians, taken in numerical order, without any selection and the average of these families, where neither parent was reported as dying under seventy years, was between ten and eleven.

Of course, such a calculation as this, based upon the records of a life insurance company alone, does not produce a fair average, because it gives undue weight to those with large families, and leaves out altogether those that have no families at all; but I believe there is a tendency, which I myself have noticed, in the applications of French Canadians to understate the number of children in the family who have died in infancy, and I think that ten to eleven is probably about a correct average family among French Canadians where neither parent dies prematurely.

Families of fifteen are very common. I, myself, knew personally one extraordinary case, a man who had had thirty-six. He had been married twice, and had eighteen by each wife. But there have been comparatively frequent cases of twenty odd children by a single wife.

It may also be of interest to know that there is still a law in the Province of Quebec which is also being practically carried out, to the effect that any man, resident in the Province who has ten living children, is entitled to get from the government a free farm of one hundred and sixty acres in the backwoods of the country, and I have heard of English speaking people, as well as many Frenchmen, who have claimed that bonus.

Those who are concerned on this side of the line with the question of race suicide, might do well to take a leaf out of the books of our French Canadian friends.

The rate of infant mortality in Quebec is heavy. Wherever you have a very high birth rate, it seems almost inevitable that there shall also be a high infantile death rate, and the death of children is not looked upon by any means as seriously as it is with us.

I understand that pressure is still brought to bear upon the people in favor of early marriages and large families, but there is no doubt

that apart from any pressure from outside altogether, the French Canadians now look upon it as a natural and proper thing to have large families. Furthermore, they have a strong race patriotism. They realize that it is only by having large families that they can hope to become very numerous or very influential on this continent, and they actually like to have large families, believing that in this way they are advancing the interest of their race.

METHODS OF INTERVALUATION ASCERTAINMENT OF RESERVES—MILES M. DAWSON.

VOL. VIII, PAGE 55.

WRITTEN DISCUSSION.

MR. GAYLORD: It is generally conceded that an annual statement of a Life Insurance Company's condition, containing such items as its reserve liability and deferred premiums calculated exactly, is as frequent a statement of such accuracy as is demanded by the nature of life insurance, and also that any intervaluation statements that may be required for presentation to the officers can show the general progress and condition of the company if such items therein are obtained by a careful and scientific interpolation from previous valuations. Especially is this true of a Company of a steady and normal growth.

However, the chief obstacle to an exact reserve item in these quarterly or semi-annual statements is the large amount of extra labor necessary to obtain it, and if any system is devised whereby this is reduced to a working basis, the obstacle disappears, and the author of the system deserves our thanks. Certainly progress should be made in the direction of greater exactness and it would seem from the consideration I have been able to give Mr. Dawson's paper that he has reduced this objection to a minimum, at least under the system of valuations generally in use.

This is chiefly apparent when we consider his mode of making reserve valuations at frequent intervals as a gradual evolution of the regular December reserve item. Thus if during each period adopted, whether it be a day, week, month or quarter, the new issue of that period with its reserve is entered promptly on the registers, as well as the reserves for the $(n + 1)^{th}$ year on the previous issues of the corresponding period in former years as they are renewed—the changes and discontinuances being also noted—a great portion of the labor for the regular annual valuation so far as the business of that period in each year is concerned has been accomplished. This will be especially the case with policies of several years standing, where the rate of discontinuance is low. In addition, the data necessary under Mr. Dawson's plan are obtained and the exact reserve liability may be ascertained at practically any time.

Mr. Dawson's method is certainly superior to any of the four he mentions as having been in use, with the possible exception of that by interpolation which, however, gives only an approximately correct

item at one or two intervaluation dates. And it is probable that in a practical application several labor-saving devices would present themselves which may not appear at first glance. To the reserve book which is outlined I would suggest an additional column giving the net change in the aggregate reserve, although of course the net difference could be interlined in red ink figures in the total "On" or "Off" column as the case might be—generally in the former. Whether the greater degree of accuracy in the reserve obtained in this manner justifies the extra labor involved would ultimately depend I should say upon the conditions existing in each office, and to this, we should remember, must be added that necessary for obtaining the deferred premiums if not the outstanding.

It has occurred to me that the card system with its tendency toward simplicity and ease of manipulation could be profitably applied not only to Mr. Dawson's suggestions, but to the ascertainment of the regular annual valuation. Cards supplanting the registers generally in use and giving similar data, with the deferred premiums at stated intervals as well, could be handled very easily, the summation being obtained by the use of the comptometer. With tabs showing the months of premium due dates, they could be arranged in such groupings as would be most convenient, the discontinuances being transferred to a separate box, new cards inserted for changed policies, and those under restorations being simply replaced in their original position. The system may already be in use in some offices, and I have not found time to follow out the idea to any extent. But I think further investigation might prove profitable.

MR. HENDERSON: There are various questions which are raised by this paper, and with regard to what might be considered the principal one, namely, the merits of the plan proposed for accomplishing the stated object, there is not very much to be said. I think the method adopted covers the various points which it is necessary to attend to in order to accomplish the stated purpose. There is a question, however, in my mind as to the advisability of any effort to accomplish this object. In all such questions as this, of expending time, which in the majority of companies means money, in obtaining information in relation to the progress of the company's business, it appears to me that there is one question which should always be expressly considered, and that is, what use is going to be made of the information when it is obtained, and how is this information going to affect the action of the company, particularly as to whether sufficient profit can be secured from such action to counterbalance the expenditure in obtaining the information. In the case of a bank there is a clear object to be gained by the daily balance sheet, as it is essential that a very close watch should be maintained on the fluctuations of the cash reserve, and immediate action can be taken in the light of any information which may be received from the balance sheet, In the case of a life insurance company, however, it does not appear that any information can be secured by a daily, or

weekly, or monthly valuation, which could not in effect be more easily secured in so far as it would affect the actions of the company, by a close watch on the various accounts which affect the profits of the company; such as mortality, interest, earnings and expenses.

Even on the assumption, however, that the object set forth is not itself of sufficient importance to justify the expenditure involved, possibly, by a slight modification, the method may be adapted to the accomplishment of an analogous object, which will repay the expenditure involved. The point which I have in mind is the prompt completion of the valuation of the company after the expiration of the calendar year, and it has occurred to me that possibly this might be accomplished by a valuation made, as of December 31, of all the policies in force at some earlier date, which might be adjusted to meet the convenience of the particular company using the method. In some companies the date fixed might be September 30, in some June 30, and, perhaps, in some December 31 of the preceding year. The method might then be applied to keep track of the effect on the total reserve of any terminations, changes, restorations, or new issues during the intervening time. The labor involved in this way would be greatly reduced, as no account whatever would require to be taken of the policies reaching their anniversaries during the interval.

There is what might be called a side issue suggested by a couple of remarks in the paper. The first paragraph on page 56 and the last sentence of the last paragraph on the same page seem to imply that the method of valuation which is presupposed as that adopted by the Company, is what is called the seriatim method, and I have noticed on a couple of occasions lately the statement made in discussions on the other side that this is the method universally adopted in this country, and perhaps the time is ripe for an experience meeting on this subject, as it would be interesting to know to what extent this statement will hold true. My own impression is that such a statement is very wide of the mark.

Oral Discussion.

Mr. Ireland: I only want to say a word in reference to the last subject touched on by Mr. Henderson. I may say that the company with which I am connected continues to make a seriatim valuation; the difference in size of the companies makes this somewhat less onerous than it would be in Mr. Henderson's office, but the great factor which seems to make it necessary for us to keep up a seriatim valuation is the tax laws. The law of Massachusetts and the laws of some other States make it necessary to get the reserve on the 31st of December on policies in force in the different States, as there is a tax based on that reserve, so that while we might be facilitated in getting our total valuation by grouping our policies, the whole thing would have to be done over again for this purpose. I may say that we are accustomed to take the figures on the 30th of September and foot them up, that is,

the valuation for December 31st on our business as it stands on the 30th of September. This gives the officers an idea of how things are moving, and some idea of what the change in reserves is likely to be at the end of the year.

MR. WEEKS: Among Mr. Henderson's valuable notes one suggestion in particular is especially interesting to me,—that of a possible method of making an annual valuation of a company in very quick time. The method he suggests is practically that which the company I am connected with has used for some years past. Our valuation is made by groups,—what we term a group being a subdivision including all the policies of the same kind of insurance, year of issue and age at issue. The amount of insurance being known in each such group, of course it can be valued as if it were one policy. During the course of the year we calculate the Reserve as at the end of the current year upon all these groups as they stood at the beginning of the year. We keep an account of all the terminations and changes of every kind during the year according to groups, noting the plus or minus effect on the Reserve of each group. A current footing is kept of these plus and minus items, and we are ready at a moment's notice to apply the net change to the total Reserve. This method is used for all issues except those of the last four years. For those four years we make a valuation upon the insurance in force on the first of December, and then keep the plus and minus account through the month of December, I may add that our company's valuations have always been on the group principle and never seriatim.

MR. PAPPS: Mr. Henderson suggested having somewhat of an experience meeting in regard to this matter, and I was about to speak before Mr. Weeks spoke on the subject. Mr. Henderson's suggestion as to valuing policies ahead of time, and his method, is one I was somewhat instrumental in putting in force in a company I was connected with some years ago, and also in the company I am now connected with. I find the one great advantage of the method of valuing ahead of time is this: we value all our policies which are in force at the beginning of the year, and we assume they are in force at the end of the year. (In a small company it is not so necessary to disregard the last three years.) We can have that valuation thoroughly checked, and from that we can calculate the expected death loss in any way we see fit. By keeping the cancels, etc., up to date, we find no difficulty in getting out the report at the end of the year.

MR. MOIR: Mr. President, a paper like the one before us is of great value, as it brings out the views of so many people on this question of valuation. At the same time I wish to emphasize a remark made by Mr. Henderson, that the subdivision into smaller intervals than a year seems to be carrying our valuation practice into too great refinement. Originally I was educated to believe that a valuation

once in five or seven years was quite sufficient, while even in five years the variation in the mortality of a small company might be considerable. Coming down to a valuation every year, which, of course, is necessary here, the fluctuations we know are still more considerable, as we found at the close of last year in the valuation of the assets side of the account; the valuation last December showed in many cases large shrinkage. In like manner the fluctuation in mortality is great. When, therefore, we subdivide the year still further and make a valuation once a month, the fluctuation may be out of all proportion, and the result is that figures brought out by valuation once a month cannot be used for any practical purpose.

One point raised by Mr. Gaylord is a very interesting one, of which practical use might be made for internal office work. I refer to the use of an adding machine or one of those recording machines for the valuation at the end of the year. I think that could be developed and worked satisfactorily. It is a subject I have thought of, but have not yet put into operation. I think it is worth our study, and for the internal valuation, the method could with advantage be used. I would not suggest anything of the kind for the State valuation records, because I still think that the primary records of an insurance company should be kept in books, as cards are liable to be mislaid; but for internal valuation purposes, I think that the card system can be used with great advantage, the recording and adding being performed by machine.

MR. J. K. GORE: Mr. President, the question of striking balances oftener than once a year has been before our company, and, for a great many years we have found our reserves approximately once a quarter and at the same time made up a balance sheet. It may be that we do unnecessary work, but we get a great many lessons from it. We find, for example, in our industrial business, the effect on the balance sheet of varying rates of mortality during the year, the heaviest mortality being in the first quarter, and the lightest in the third quarter. The fact that the third quarter of the year, in the ordinary business, as a rule, shows the lowest increase in premium income, while the increase in reserve in that quarter is approximately normal tends to diminish the increase in surplus for that particular period. A number of other facts and conditions are thus brought to light that we would miss altogether if we had our valuation but once a year.

NOTE ON LUBBOCK'S FORMULA—JOHN A. McKELLAR.

VOL. VIII, PAGE 58.

WRITTEN DISCUSSION.

MR. SHEPPARD: Mr. McKellar's proof of Lubbock's Formula really consists of two separate parts; first, the expansion of $\frac{x}{(1+x)^t - 1}$ in a series of ascending powers of x which is much easier than the proof given in the Text Book. Secondly, an application of the method of symbolic operators to find $\Sigma u_{\frac{x}{n}}$. This, it seems to me, is a good opportunity to bring together the formulas connecting the different operators and make one or two deductions from them. Using D and Δ in the usual sense for a difference of unity in the suffix, and d and δ for a difference of $\frac{1}{n}$ in the suffix, we have

$$\begin{aligned} \mathrm{D} &= 1 + \Delta \\ d^n &= \mathrm{D} \\ \therefore\ d &= (1+\Delta)^{\frac{1}{n}} \\ \text{and } \delta &= d - 1 = (1+\Delta)^{\frac{1}{n}} - 1. \end{aligned}$$

Now, $\delta \Sigma_o^{mn-1} u_{\frac{x}{n}} = u_m - u_o$

$$\therefore \Sigma_o^{mn-1} u_{\frac{x}{n}} = \frac{1}{\delta}(u_m - u_o) = \frac{1}{(1+\Delta)^{\frac{1}{n}} - 1}(u_m - u_o) \quad (\mathrm{A})$$

$$= \frac{\Delta}{(1+\Delta)^{\frac{1}{n}} - 1} \cdot \frac{1}{\Delta}(u_m - u_o) \quad (\mathrm{B})$$

(B) is Mr. McKellar's formula; and since from either Chapter XXIII, Section 39, or Chapter XXIV, Section 39, we have the following additional formulas

$$\frac{d}{dx} = \log.(1+\Delta) = \log. \mathrm{D} = n \log. d$$

$$\text{or } 1 + \Delta = e^{\frac{d}{dx}}$$

(A) becomes $\dfrac{1}{e^{\frac{1}{n}\frac{d}{dx}} - 1}(u_m - u_o)$

which, making use of the expansion in Chapter XXIV, Section 39 becomes

$$n\int_o^m u_x\,dx - \frac{1}{2}(u_m - u_o) + \frac{1}{12n}\left(\frac{du_m}{dx} - \frac{du_o}{dx}\right) - \frac{1}{720n^3}\left(\frac{d^3u_m}{dx} - \frac{d^3u_o}{dx}\right) + \text{etc.}$$

which is Woolhouse's formula.

This is another way of showing the connection between Woolhouse's and Lubbock's formulas.

Again, some of the equations in Chapter XXIII can be simplified by the use of operators, e. g.

$$\frac{d^2}{dx^2} = (\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \text{etc.})^2 = \Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 + \text{etc.}$$

which is formula (15)

$$\frac{d^3}{dx^3} = (\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \text{etc.})^3 = \Delta^3 - \frac{3}{2}\Delta^4 + \text{etc.} \quad (16)$$

and since $\frac{d}{dx} = \log.\ (1 + \Delta) = \frac{1}{m} \log.\ (1 + \Delta')$

where Δ' represents the difference of m in the suffix

$$\therefore \frac{d}{dx} = \frac{1}{m}(\Delta' - \frac{1}{2}\Delta'^2 + \frac{1}{3}\Delta'^3 - \text{etc.})$$

which is formula (19)

Similarly (21) and (22) come at once.

Again referring to Chapter XXIII, Sections 16 to 22 inclusive a very elaborate proof is given of formulas to be used in getting subdivided differences in terms of differences for difference of unity in the suffix, these formulas come at once by expanding $(1 + \Delta)^{\frac{1}{n}} - 1$ in terms of Δ and then raising the resulting expansion to the successive powers up to the fifth.

Address of the President,

ISRAEL C. PIERSON, PH. D.,

at the opening of the Autumn Meeting, October 20th, 1904.

Gentlemen of the Actuarial Society of America:

It is pleasing that so many of the members are convened at this meeting in response to the invitation of our good friends, the officers of the Penn Mutual Life Insurance Company and the Provident Life and Trust Company. Eleven years ago yesterday, the tenth semi-annual meeting of the Society was held in this historic city under these same kind auspices and it is interesting to observe that such a good portion of those who attended that meeting, October 19th and 20th, 1893, are now present. Of that company of forty-three we have lost seven, but we hold them in cherished memory, Sheppard Homans, George Ellis, Augustus F. Harvey, William McCabe, Horace C. Richardson, Henry W. Smith and William E. Starr,—one ex-President, four past-Councillors and two members.

This is the thirty-second meeting of the Society, counting in the list the Fourth International Congress of Actuaries held in New York, August 31st to September 5th, 1904, an account of which is contained in No. 30 of *Transactions*, and the recollections of which will long be bright in memory.

Mr. St. John, in his address as President, at the meeting of October, 1893, spoke of the conditions of the Society at that time and mentioned the "Visible signs of prosperity—a liberal and increasing membership, individual zeal in respect of contributions to its proceedings, an increasing intimacy among its members in professional and personal relations." The membership consisted of the founders and those "connected with actuarial pursuits" who had been recommended by the Council and elected by the Society.

Mr. Fackler in his President's address at Toronto, September 30th, 1891, suggested a classification of members. The subject was referred to the Council. At the following meeting of the Society, April 28th, 1892, Mr. McClintock, as Chairman of a Special Committee of the Council, reported amendments to the Constitution to provide for the enrollment of Associates and admission to membership by examination. These proposed amendments were discussed at two meetings but were finally withdrawn at the annual meeting, April 28th, 1893. The matter was again brought to the attention of the Society by Mr. McClintock at the Montreal meeting, October 10th, 1895. At the annual meeting in New York, April 30th, 1896, Mr. McClintock devoted his entire address to the subject and said: "The present meeting will be a memorable one, should the Society vote affirmatively upon the proposed amendments to the Constitution. If the amendments are adopted a great change takes place affecting the organization of the Society through all future time." The Amendments to the Constitution which were very similar to those proposed four years previously were adopted at that meeting. Under that change in the Constitution the first enrollment of Associates was reported to the Society by the Council at the meeting of October 7th, 1897, six years after the matter was first suggested. Since then a large number have been enrolled as Associates, many of whom have been admitted to membership on examination. There has been a constant and a healthy growth, and many men have been added to the roll who will be the strength of the Society in the future. On October 29th, 1893, eleven years ago, the membership was seventy. Now it is:

Members	122
Associates	34
Total	156

On account of this increase in the membership it seems desirable to enlarge the elected membership of the Council. For this purpose Amendments to the Constitution have been proposed, constitutional notice of which has been given to the members. You will vote upon these amendments at this meeting.

Philadelphia is indeed an historic city. It is said that the first settlement of Europeans within what are now its precincts was

made in the year 1636, nearly three centuries ago, long enough to have had happen in it many things of national and international importance. Independence Hall stands as a memento of the Declaration on July 4th, 1776, "that these United Colonies, are and of right ought to be free and independent States." It was here that the first flag of the United States—the five-pointed white stars on a blue ground and the alternate red and white stripes—was devised and made by Betsy Ross. I believe that I utter the sentiment of every American here whether he lives in Canada or the States, that one's heart thrills with joy and a feeling of peaceful security to see here the British Flag and the Stars and Stripes intertwined. Any one of us roving this wide world on land or sea would feel safe and happily content if either emblem were floating above him. In his first address as President, Mr. Macaulay remarked, "In the Actuarial Society of America, Americans and Britons are not merely cousins but brethren." It is quite appropriate that these national colors should be the decorations of this hall and on this occasion, for we are in Φιλαδελφία.

This is not the time to catalogue the important events which have occurred in this great city, called the "Cradle of Liberty." Just now it is interesting to us because it is the "cradle of insurance" on this continent, for here were established the first companies. In Fire Insurance "The Philadelphia Contributionship" was founded in 1752. In Life Insurance "The Presbyterian Ministers' Fund" was started in 1759. In Marine Insurance "The Insurance Company of North America" began business in 1792.

We all share a sense of deep sorrow, and a feeling of great loss to the Society on account of the death on May 26th, 1904, of our fellow member, Dr. Charlton T. Lewis. His paper at the May meeting was the last and one of the best of his contributions to the Society's *Transactions*. He was a ripe scholar, a subtle thinker, a versatile writer, a ready and instructive speaker, a courteous gentleman, a kind friend, and an honor to the Society.

Abstract from the Minutes of the Regular Autumn Meeting of the Actuarial Society of America, held in Philadelphia, Pa., on Thursday and Friday, October 20th and 21st, 1904.

FIRST SESSION.

Bellevue-Stratford Hotel,
Philadelphia, Pa.,

October 20th, 1904, 11:00 A. M.

The thirty-second meeting of the Society was called to order by the President, Mr. Pierson.

MEMBERS PRESENT.

Alsop,	Hunter, A.	Sheppard,
Barker,	Ireland,	Smith,
Craig,	Lee,	Tatlock,
Crawford,	Linzmeyer,	Torrey,
Fackler,	McClintock,	Van Cise,
Field,	Macaulay,	Weeks,
Gaylord,	Messenger,	Welch,
Gore, J. K.	Moir,	Wells,
Graham,	Nichols,	Wing,
Hall, C. C.	Ogden,	Wright, P. T.
Hardcastle,	Papps,	Wright, W. C.
Henderson,	Pierson,	

ASSOCIATES ATTENDING.

Brown,	Davenport,	Gibb,
Bulkley,	Dickenson,	Hammond,
Cochnower,	Dow,	Jackson,
Craig,	Forster,	Morris.

After the calling of the roll, the President addressed the Society. (See page 179.)

Mr. Alsop and Mr. Barker then announced to the Society the hospitable programme that had been arranged by the officers of the Provident Life and Trust Company and the Penn Mutual Life Insurance Company for the entertainment of the members during their sojourn in Philadelphia.

The minutes of the meeting of the Society held in New York on May 19th and 20th, 1904, were approved as printed with *Transactions*, Vol. VIII., No. 31.

The Secretary reported on behalf of the Council, that Mr. Tatlock, the Secretary, having asked to be relieved from office, his resignation had been accepted with regret, and Mr. Arthur Hunter had been appointed Secretary *pro tempore*. Also that the Council recommended that the Secretary be permanently relieved of the duty of editing the *Transactions*, and that an Editor be appointed; and, pending the adoption of an amendment to the Constitution creating such an office, the Council had requested Mr. Clayton C. Hall to act as Editor.

The President referred to the death of the late Dr. Charlton T. Lewis, whereupon Mr. McClintock presented a minute that he had prepared in memory of Dr. Lewis, which, upon motion of Mr. Weeks, seconded by Mr. C. C. Hall, was adopted, and the Secretary was instructed to send a copy to the family of Dr. Lewis. (See page 186.)

The next business in order was the consideration of the proposed amendments to the Constitution increasing the number of Members of the Council to be elected, in addition to the Officers and past-Presidents, Members *ex-officio*, from six to nine, and in respect to quorums. After some discussion a vote was taken by show of hands, which resulted in fourteen affirmative votes and nine opposed. The amendments were pronounced lost for lack of a constitutional majority.

The Society then listened to the reading of papers presented at this meeting. (See pages 105–134.)

At 12:40 P. M. the Society adjourned until 2:30 P. M., and during the recess were entertained at luncheon in the hotel by the officers of the Provident Life and Trust Company and the Penn Mutual Life Insurance Company.

SECOND SESSION.

THURSDAY, OCTOBER 20th, 1904, 2:30 P. M.

The meeting was called to order by the President, and the Society entered upon the discussion of papers presented at the previous meeting in May, 1904. (See pages 135–178.)

On motion of Mr. Smith, seconded by Mr. Welch, the vote by which the proposed amendments to the Constitution was lost was reconsidered, and after some further discussion, the question being upon their adoption, they were adopted by a vote of twenty-five ayes and three nays.

The amendments are as follows:

Article V of the Constitution is amended as follows: In the first line of said Article the word "nine" is substituted for the word "six." And in place of the second sentence of said Article the following is substituted: "At the first election of Members of Council after the adoption of this Article, a number of members shall be elected sufficient to make up three classes of three each to serve respectively for three years, two years and one year; and annually thereafter three members shall be elected for three years."

Article XI of the Constitution is amended as follows: In the first line of said Article the word "seven" is substituted for the word "four," and the word "twenty" for the word "ten."

On motion the meeting adjourned to reassemble for dinner at seven o'clock.

THURSDAY, OCTOBER 20th, 1904, 7:00 P. M.

Members and Associates of the Society met together for dinner at the Bellevue-Stratford Hotel. The President of the Society presided, with officers of the Provident Life and Trust Company and of the Penn Mutual Life Insurance Company occupying seats at his right and left. The health was proposed of the President of the United States and the King of Great Britain and Ireland.

Addresses of welcome to the Society were made by Mr. Samuel R. Shipley, President of the Provident Life and Trust Company, and by Mr. Henry C. Lippincott, Superintendent of Agencies of the Penn Mutual Life Insurance Company.

Brief addresses were then made in accordance with the following programme:

The Professional Responsibility of the Actuary,	Mr. Clayton C. Hall.
The Unexpected,	Mr. Archibald A. Welch.
The One Pure Science,	Mr. Walter S. Nichols.
The Future,	Mr. James M. Craig.
Life Insurance Conditions in the South,	Mr. John K. Gore.

THIRD SESSION.

FRIDAY, OCTOBER 21st, 1904, 10:00 A. M.

The Society was called to order by the President, and resumed the discussion of papers read at the preceding meeting in May, 1904. (See pages 135–178.)

On motion of Mr. Wells, seconded by Mr. Van Cise, the following resolution was adopted:

Resolved, that this Society expresses its hearty thanks to the Penn Mutual Life Insurance Company and the Provident Life and Trust Company, and to their officers, for their hospitality, and kind attentions, extended to us, officially and personally, from the first moment of our arrival in this City of Brotherly Love.

The President directed that a copy of the resolution be sent by the Secretary to each of the companies named.

At 12:30 P. M. the thirty-second meeting of the Actuarial Society adjourned *sine die*.

The Members and Associates, with the ladies accompanying them, were then entertained at luncheon by the officers of the two Philadelphia companies which had been their hosts on the day before, and at two o'clock were taken upon an enjoyable drive through the public park, stopping on the way at the City Hall, where they were received by His Honor, the Mayor of Philadelphia.

ARTHUR HUNTER,
Secretary *pro tempore*.

ERRATA.

(Correction furnished by Mr. C. D. Higham.)

Vol. VII, page 540, line 5: for "parentheses," read "curved brackets."
" " line 19: for "color or a blank) would," read "color), or a blank, would."

CHARLTON THOMAS LEWIS died on May 26, 1904, in his seventy-first year. His years did not diminish his usefulness to this Society, which recognized him as its most broadly accomplished member.

Mr. Lewis was graduated at Yale College in 1853, where he was chosen as poet of his class. A few years later he was for a time Professor of Mathematics in Troy University, and his interest in mathematical topics throughout life was illustrated by his membership in the American Mathematical Society. Adopting the law as a profession, he was more particularly devoted to the law of insurance. In 1873 he took part, as secretary, in the organization of the Chamber of Life Insurance, and remained until its dissolution in 1878. The chamber was in a sense a forerunner of this Society, although composed of companies instead of individuals, and organized for practical rather than professional objects. Its chief work of permanent value was the collection of mortality statistics from thirty American life companies, a work in which the secretary was warmly interested. He was a pioneer in the movement to introduce insurance as a subject of university education, and delivered lectures at Columbia, Cornell and Harvard Universities. He received from Harvard the honorary degree of Doctor of Laws, having previously received elsewhere that of Doctor of Philosophy. His latest work was an important paper on the Rate of Interest, which he read before this Society a few days before his death.

His interests and activities beyond the scope of the Society can only be glanced at here. He touched nothing that he did not improve and adorn. He was a constant member of a small club which met weekly for forty years for personal improvement in the knowledge of Greek. He wrote the great work known as Harper's Latin Dictionary, as well as a history of Germany, and other books. He was versed in various modern languages, giving, for instance, much time to the critical study of Dante. He wrote and spoke his own tongue clearly and eloquently. On the only occasion of his active intervention in politics, at the Gold Democratic Convention of 1896, at Indianapolis, his was the most effective speech. His voice and pen were never refused to any worthy cause. For many years, and until his death, he served as President both of the New York Prison Association and of the State Charities Aid Association of New Jersey. The Society records its tribute to the memory of a brilliant and beloved member, not more admired and honored by his fellow members for his intellectual gifts and achievements than for the gracions charm and courtesy of his personality.

INDEX TO VOLUME VIII.

PAGE.

BIRTH RATE IN NEW SOUTH WALES, THE DECLINE IN THE. Richard Teece 43

BURRAGE, ARTHUR FRANCIS
Obituary 103

CRAIG, J. M.
Remarks 81, 139

CRAWFORD, J. C.
Remarks 135

DAWSON, M. M.
Methods of Intervaluation Ascertainment of Reserves 55

DISCUSSION OF PAPERS READ AT MEETING MAY 20TH AND 21ST, 1903 65–89
Mortality, Extra, considered in connection with deferred dividend contracts. H. W. Robertson 65
New Valuation Formula. M. M. Dawson 67
Method of finding a Single Life equivalent to several Joint Lives by adjusting Tables following Makeham's Law. D. H. Rose 84
Mortality Rate of the City of New York, Investigation into. J. F. Roche 86
Reserves on Insurances under which a Super-Normal Mortality is expected. A. Hunter 88
Specialized Mortality Investigation. Committee's Report 89

DISCUSSION OF PAPERS READ AT MEETING MAY 19TH AND 20TH, 1904 135–177
Maximum Single Risk, etc. W. S. Nichols 135
A Factor of the Rate of Interest. C. T. Lewis 148
Experiment with the Specialized (Mortality) Investigation. R. W. Weeks 154
Frequency Curves and Moments. R. Henderson 159
Decline in Birth Rate in New South Wales. Richard Teece 164
Intervaluation Ascertainment of Reserves. M. M. Dawson 172
Note on Lubbock's Formula. J. A. McKeller 177

ERRATA 185

FACKLER, D. P.
Remarks 71, 78, 80, 83

FACKLER PRIZE 96, 97

FREQUENCY CURVES AND MOMENTS. Robert Henderson 30

GAYLORD, W. S.
Remarks 84, 172

GORE, J. K.
Remarks 87, 92, 176

PAGE.

HALL, C. C.
Remarks..... 75, 148

HANN, R. G.
Remarks..... 73, 82, 86, 164

HENDERSON, ROBERT.
Frequency Curves and Moments..... 30
Remarks..... 88, 143, 163, 173

HUNTER, A.
A Graduation of the Specialized Mortality Experience..... 114
Remarks..... 89

HUTCHESON, W. A.
Remarks..... 65

INTERVALUATION ASCERTAINMENT OF RESERVES, METHODS OF. M. M. Dawson..... 55

IRELAND, O. B.
Remarks..... 150, 174

LEWIS, CHARLTON T.
Notes on a Factor, hitherto overlooked, of the Rate of Interest..... 8

LEWIS, CHARLTON THOMAS.
Obituary..... 186

LUBBOCK'S FORMULA, NOTE ON. John A. McKellar..... 58

MCCLINTOCK, E.
Remarks.....76, 80, 145

MCKELLAR, JOHN A.
Note on Lubbock's Formula..... 58

MACAULAY, T. B.
Remarks.....69, 74, 83, 144, 166, 169

MAXIMUM SINGLE RISK AND THE ACCEPTANCE OF EXCEPTIONAL CLASSES OF RISKS, ON THE PRINCIPLES WHICH SHOULD DETERMINE THE. Walter S. Nichols..... 1

MEIKLE, JAMES.
Obituary..... 102

MILLER, B. J.
Remarks..... 71, 82

MINUTES OF MEETING:
May 19th and 20th, 1904..... 95
October 20th and 21st, 1904..... 182

MOIR, H.
Remarks.....73, 84, 175

NICHOLS, WALTER S.
On the Principles which Should Determine the Maximum Single Risk and the Acceptance of Exceptional Classes of Risks..... 1
Remarks.....78, 146, 150

NOTATION..... viii

PAGE.

OBITUARY NOTICES :
John Bell Tennant 101
James Meikle 102
Arthur Francis Burridge 103
Charlton Thomas Lewis 186
OFFICERS, MEMBERS AND ASSOCIATES ..... iv–vii
OGDEN, S. N.
Remarks 137
PAPPS, P. H. C.
Remarks 175
PIERSON, ISRAEL C.
Address as President, May 19, 1904 60
Address as President, October 20, 1904 179
RATE OF INTEREST, NOTES ON A FACTOR, HITHERTO OVERLOOKED, OF THE. Charlton T. Lewis 8
REPAYMENT OF LOANS BY CONTINGENT INSTALMENTS, ON THE. H. W. Robertson 128
ROBERTSON, H. W.
On the Repayment of Loans by Contingent Instalments 128
SANDERSON, F.
Remarks 76
SHEPPARD, H. N.
Remarks 68, 82, 142, 177
SPECIALIZED (MORTALITY) INVESTIGATION, AN EXPERIMENT WITH THE. R. W. Weeks 17
SPECIALIZED MORTALITY EXPERIENCE, A GRADUATION OF THE. A. Hunter 114
TEECE, RICHARD.
The Decline in the Birth Rate in New South Wales 43
TENNANT, JOHN BELL.
Obituary 101
TOTAL ABSTINENCE, EFFECT OF, ON THE DEATH RATE. J. G. Van Cise 105
VAN CISE, J. G.
Effect of Total Abstinence on the Death Rate 105
Remarks 74, 83, 154
WEEKS, R. W.
An Experiment with the Specialized (Mortality) Investigation 17
Remarks 66, 74, 175
WELCH, A. A.
Remarks 152
WELLS, D. H.
Remarks 67, 82, 83, 146, 157
WRIGHT, W. C.
Remarks 165

HG
8754
A4
v.8

Actuarial Society of
America
Transactions

PLEASE DO NOT REMOVE
CARDS OR SLIPS FROM THIS POCKET

UNIVERSITY OF TORONTO LIBRARY